Design of Multilevel Codes

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I. INTRODUCTION

For 2^k-ary digital transmission schemes like ASK, PSK, QAM etc. channel coding and modulation can be combined via set partitioning and multilevel coding (MLC) [4] in an efficient way. Often, maximizing the minimum squared Euclidean distance (MSED) for sequences of signal points was proposed as an optimum design rule for MLC schemes. This approach balances the products $d^2_i \cdot \delta_i$ of MSED $d^2_i$ in the i-th level signal subset and the minimum Hamming distance $\delta_i$ of the individual component code $C_i$. Codes with excellent asymptotic gains may be constructed by this design rule.

MLC can be decoded efficiently by multistage decoding (MSD), i.e. each component code is decoded individually starting from the lowest level and using decisions of prior stages. Unfortunately, the performance of MLC schemes is far below asymptotic promises derived from Euclidean distances. Errors at the lowest level clearly predominate and cause a lot of additional errors at higher levels via error propagation in MSD. Often, the suboptimum MSD procedure erroneously is assumed to cause this deficiency. But we show that a tremendous increase in the number of nearest neighbour error events due to multiple representation of binary symbols is responsible for the degradation. For this reason, balanced distances $d^2_i \cdot \delta_i$ turn out to be a quite improper design approach for MLC.

In this paper, a new design rule is derived from information theory. We show that, applying the new design rule, MLC together with MSD is, in principle, an optimum approach to combine channel coding and modulation.

II. NEAREST NEIGHBOUR ERROR EVENTS

At low levels binary symbols $c^i$ are represented multiply by signal points. For instance, for signal points taken from the $D$-dimensional lattice $Z^D$ the inverse binary symbol is represented by $2D$ nearest neighbours points. Thus, a single binary codeword of $C^i$ with minimum weight $\delta_i$ causes up to $(2D)^i \delta_i$ sequences of signal points with MSED $d^2_i \cdot \delta_i$ from a sequence representing the all-zero-codeword. As the highest minimum Hamming distance is required at level $i = 0$ the highest degradation due to the increased number of nearest neighbour error events is observed at this level. This explosive increase of nearest neighbour error events has to be taken into account for the rate design of multilevel codes.

In general, for linear binary component codes $C^i$ the average number of error events with MSED $d^2_i \cdot \delta_i$ is increased by the factor $(N_{Z_i})^{1/2}$. Where $N_{Z_i}$ denotes the average number of nearest neighbour signal points representing the inverse binary symbol in the subsets at the i-th partitioning level [2].

III. CAPACITIES OF EQUIVALENT CHANNELS

At each level i equivalent channels can be considered for transmission of binary symbols $c^i$. The sum of the capacities $C^i$ of these equivalent channels yields the capacity $C$ of the communication scheme [2, 5]. The capacity $C$ can be achieved via MLC and MSD if and only if the individual rates $R^i$ at different coding levels are chosen equal to the capacities $C^i$ [2].

Thus, designing the rates $R^i$ from capacity arguments, MLC together with MSD is an optimum way for coded modulation. Furthermore, the use of binary codes is optimum in principle for channel coding for any digital modulation scheme.

For codes with small length $n$, we propose a rate design according to the isomorphisms of the random coding exponents $E_i^c(R^i)$ for the equivalent channels at level $i$ [3];

$$E_i^c(R^i) = \left( -\log_2 p_e \right)/n = \text{const}.$$  \hspace{1cm} (1)

Where $p_e$ denotes the probability of block error.

Compared to the design for balanced distances, the rates $R^i$ at low levels due to the new design rule [2] have to be chosen smaller in order to compensate for the effect of an increased number of nearest neighbour error events.

IV. APPLICATIONS

Digital PAM transmission with MLC for the AWGN channel was investigated by simulations. Turbo-codes [1] with rates equal to the capacities $C^i$ are employed as component codes. For 8PSK with total rate $R = 2$ and blocklength $n = 2000$ a bit error rate (BER) of about $10^{-5}$ is achieved only 1.4 dB above capacity. In contrast to the design rule for balanced distances, here, error rates at all levels are quite balanced. For more bandwidth efficient communication schemes like 16QAM with $R = 3$, 32QAM with $R = 4$ and 64QAM with $R = 5$ results are analogous. Using longer codes with $n = 20000$ and e.g. 16QAM with $R = 3$, a BER of about $10^{-5}$ only 0.8 dB above capacity has been observed. Furthermore, high uncoded levels in QAM schemes can be employed to achieve an additional shaping gain.

V. CONCLUSIONS

The design rule for balanced distances leads to unsatisfactory results due to an enormous increase in the number of nearest neighbour error events. In contrast, rate design according to random coding exponents or capacities of the equivalent channels permits to make use of the benefits of binary codes in any digital transmission scheme. Application of Turbo codes to MLC schemes results in digital communication close to capacity with a wide range of trade-off between power and bandwidth efficiency. Similar to the theoretical separation of source and channel coding, binary channel coding and modulation can be treated and optimized separately if they are combined via MLC.

REFERENCES


