Lattice-Reduction-Aided Tomlinson-Harashima Precoding for Point-to-Multipoint Transmission

Clemens Stierstorfer and Robert F. H. Fischer

Abstract  A new version of Tomlinson-Harashima Precoding (THP) for multiple-input/multiple-output (MIMO) channels with decentralised receivers employing lattice reduction is introduced. The proposed method clearly outperforms linear equalisation and conventional THP and in particular shows full diversity order. The proposed method clearly outperforms linear equalisation and conventional THP and in particular shows full diversity order.

Keywords  MIMO, point-to-multipoint transmission, precoding, lattice reduction

1. Introduction

A central transmitter serving several geographically scattered receivers is a common scenario in (wireless) communication. Since joint processing of the received signals is impossible, solely transmitter based techniques for the avoidance of interference due to parallel transmission of the users’ signals can be applied.

In this letter, we present a version of the well-known Tomlinson-Harashima Precoding (THP), that outperforms prior approaches in terms of symbol error rate (SER).

2. Transmission Model and Formulation of Problem

We consider transmission using $N_T$ antennas at the base station (BS) to $K \leq N_T$ mobile stations (MS) scattered around the BS, each equipped with a single receive antenna. Our derivations are restricted to frequency-flat channels, i.e., no signal dispersion is imposed.

Employing an advantageous all real-valued vector-matrix notation [1], the receive signal is given by the common MIMO model

$$ y_r = H_r x_r + n_r, $$

where $\{\cdot\}_r$ indicating real quantities. The vectors are obtained from stacking real and imaginary parts of $y := [y_1, \ldots, y_K]^T$ containing the receive signals of each MS, $x := [x_1, \ldots, x_{N_T}]^T$ the vector of transmit signals and $n := [n_1, \ldots, n_K]^T$ representing the additive, white Gaussian noise with variance $\sigma_n^2$. The real-valued channel matrix is obtained from $H := [h_{kl}]$, with the complex gain factors $h_{kl} \sim CN(0, 1)$, i.e., flat-fading channels are present between each pair of antennas. We assume the channel matrix to be constant over a certain time, and then independently change to another realization (block fading, see e.g. [2]). Furthermore, in order to show ultimate performance curves channel state information (CSI) is assumed to be perfectly known at the transmitter (CSI might be obtained from the reverse link in TDD schemes or via a feedback channel. In case of continuously time-varying fading channels, without significant loss residual interference due to (slightly) outdated CSI could be suppressed by means of residual linear equalisation at the receiver [3]).

Note, although the $K$ received signals are comprised in a single vector, the scattered nature of the MSs prevents joint processing. I.e., only diagonal operators can be applied to $y$.

Precoding schemes generate transmit symbols $x$ from data symbols $a$ ($M$-QAM with $a_k \in \{\pm \frac{1}{2}, \ldots, \pm \sqrt{\frac{M-1}{2}}\}$), such that the channel $H$ suffers from a boost in transmit power. Tomlinson-Harashima Precoding (THP) clearly outperforms linear precoding [4], but symbol error rate curves obtained from THP still show only diversity order of $N_T - K + 1$, which in case of $N_T = K$ that of a single Rayleigh-fading channel. In the following, we present a modified technique exploiting the full diversity of the MIMO channel.

3. Lattice-Reduction-Aided Tomlinson-Harashima Precoding

3.1 Derivation

The new version of THP is based on lattice reduction. Performing a lattice reduction algorithm (e.g. the LLL algorithm [5]) of the transposed real-valued channel matrix $H_r^T$, we can write

$$ H_{\text{red}} = RH_r, $$

where $H_{\text{red}} \in \mathbb{R}^{2K \times 2N_T}$ is a reduced basis of the lattice generated by the rows of $H_r$ and $R \in \mathbb{Z}^{2K \times 2K}$ is a unimodular matrix (matrix with integer entries and determinant $\pm 1$).

Then the application of the V-BLAST algorithm [6] to the transposed reduced channel matrix $H_{\text{red}}^T$ results in the factorisation

$$ B = PH_{\text{red}}W^T. $$

Here, $B$ is a lower-left triangular matrix with unit main diagonal, $P$ is a permutation matrix and $W$ is a matrix with orthogonal rows.

In summary, we can express the (pseudo) inverse of the channel matrix as

$$ H_r^{-1} = W^T B^{-1} P R, $$

i.e., a cascade of these matrix filters would linearly equalise the channel.

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Dipl.-Ing. C. Stierstorfer and Priv.-Doz. Dr.-Ing. R. F.H. Fischer, Institute for Information Transmission, University of Erlangen-Nuremberg, Cauerstr. 7, D-91058 Erlangen, Germany. Phone: +49 (9131) 852-8718, Email: {clemenss,fischer}@LNT.de
3.2 Discussion

The new version of THP as depicted in Fig. 1 preequalises the channel in two successive steps. First, the reduced part of the channel matrix $H_{\text{red}}$ is preequalised with THP based on the matrix decomposition (3) and using scalar modulo reductions into the interval $[-\sqrt{M/2}, \sqrt{M/2})$ (comprised by the dashed rectangle in Fig. 1). Hence, from (2), it can be seen that $R^{-1}$ remains to be equalised. This is done linearly by $R$ preceding the THP structure. The receiver does not require any changes compared to conventional THP; data symbol estimates are obtained from the received signal via modulo reduction and threshold decision.

Interestingly, this precoding strategy can be identified as the dual to lattice-reduction-aided detection for decentralised transmitters as described in [7].

In classical THP the data symbols $a$ are contained in the Voronoi region of the precoding lattice $\sqrt{M}Z_{2K}$, which is defined by the modulo operation used at transmitter and receiver. Since $R$ is unimodular matrix the data symbols drawn from the Voronoi region are transformed to lie within a fundamental region of the precoding lattice. THP inherently periodically extends the initial constellation; this, however, also results in a covering of the entire space when the signals lie within a fundamental region rather than the Voronoi region. Anyway, the transmit signal $\tilde{x}$ always originates from the Voronoi region and thus has least average power.

4. Numerical Results

Results from numerical simulations for broadcast schemes with $N_T = 4$ antennas at the transmitter and $K = 4$ receivers, employing 4-QAM and 16-QAM are depicted in Fig. 2 and Fig. 3, respectively. The feedforward matrix $W^T$ is normalised for fixed short-term power; this scaling is compensated by automatic gain controls at the receivers.

The error rate curves clearly show the superiority of lattice-reduction-aided THP (LR-THP) over conventional THP and linear preequalisation (LE). In particular, the curves for LR-THP show the full diversity order of 4, and hence have the same slope as a maximum-likelihood signal generation [8] at the transmitter would offer.
References


