Widely Linear Receivers for Space–Time Block–Coded Transmission Over Frequency–Selective Fading Channels

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Abstract

In this paper, we develop receiver concepts for transmission with space–time block codes (STBC’s) over frequency–selective fading channels. The focus lies on Alamouti’s space–time block coding scheme, but the results may be generalized to other STBC’s. We show that a straightforward combination of conventional equalizers and a space–time block decoder is only possible if at least as many receive antennas as transmit antennas are employed, but not for the practically interesting case of pure transmit diversity for which space–time coding has been originally developed. This restriction is circumvented by our approach. Here, the structural properties of the transmit signal of space–time block coding, which is shown to be improper (rotationally variant), are fully utilized. For this, equalizers with widely linear (WL) processing are designed, i.e., a widely linear equalizer (LE) and a decision–feedback equalizer (DFE) with widely linear feedforward and feedback filtering. These schemes are especially suited for equalization of high-level modulated signals, which are used in third–generation time–division multiple access mobile communications standards such as EDGE (Enhanced Data Rates for GSM Evolution). It is shown that the standard detector for Alamouti’s STBC may be also viewed as a special case of a widely linear detector. Simulation results demonstrate that the proposed concepts may be successfully employed in an EDGE receiver, especially for pure transmit diversity. Here, significant gains can be observed compared to a conventional single–input single–output (SISO) transmission.

1 Introduction

Space–time coding (STC), e.g., [1, 2, 3], has been introduced in order to be able to combat fading via diversity also in such situations where multiple transmit antennas but only a single receive antenna can be employed. This often is the case for transmission from a base station to mobile stations. In literature, space–time trellis codes (STTC’s), e.g., [2], and space–time block codes (STBC’s), e.g., [1, 3], have been proposed for systems with transmit diversity. Both classes of codes have been originally designed for flat fading scenarios.

In contrast to this, only little is known with respect to suitable detection strategies for space–time codes transmitted over frequency–selective fading channels.
producing intersymbol interference (ISI). For STTC’s, an iterative receiver consisting of separate optimum soft-output channel equalization and STTC decoding blocks is presented in [4]. The optimum receiver strategy is combined maximum-likelihood sequence estimation taking into account ISI and space-time trellis coding jointly [5]. Both approaches entail a complexity far too high for transmission with high-level modulation which is used in advanced time-division multiple access (TDMA) mobile communications standards such as GSM/EDGE (Enhanced Data Rates for GSM Evolution), where 8PSK modulation is applied. On the other hand, highly power efficient reduced-state variance of the receivers of [4, 5] cannot be constructed for general STTC’s if one has to cope with nonminimum-phase channels, cf. [4]. One exception is delay diversity [6], which is a special case of STTC’s. Here, the cascade of space-time encoder and channel corresponds to an equivalent channel with one additional tap. Hence, conventional reduced-state equalization methods can be used, cf. also [7].

For STBC’s, a direct approach for successive equalization and block decoding has been given in [8] which, however, is not applicable if more transmit than receive antennas are used. In this paper, we focus on transmission with STBC’s over fading ISI channels. In general, suboptimum receivers have to be employed for complexity reasons. We show that space-time block coding generates an improper (rotationally variant) [9] transmit signal, i.e., a signal with nonvanishing pseudocorrelation. Hence, widely linear equalization is recommendable, which has been recently introduced in a different context [10]. A widely linear equalizer (LE) and a decision-feedback equalizer (DFE) employing widely linear processing are described. In contrast to these equalization concepts, conventional suboptimum equalizers cannot yield a high performance for space-time block-encoded transmission without receive diversity.

The GSM/EDGE system is considered as a potential application for the novel receivers. For the relevant GSM/EDGE channel profiles, remarkable gains can be observed compared to the case of conventional transmission with a single antenna.

2 System Model

In the following, the equivalent complex baseband representation of space-time block-encoded transmission over a frequency-selective fading channel is described. Here, we restrict ourselves to Alamouti’s space-time block coding scheme [3] for most derivations for the sake of simplicity. Nevertheless, the presented receiver concepts can be generalized in a straightforward manner to arbitrary STBC’s, cf. e.g., [11]. In Alamouti’s scheme, \( N_T = 2 \) transmit antennas are employed. Two independent, identically distributed (i.i.d.) sequences \( c_1[\mu], c_2[\mu] \) of real- or complex-valued coefficients taken from an \( M \)-ary signal set \( C \) (e.g. \( C = \{ \pm \sqrt{\frac{1}{M-1}} \} \) for binary phase-shift keying (BPSK) or \( C = \{ \frac{1}{\sqrt{P}} e^{i2\pi n} | n \in \{ 0, 1, \ldots, 7 \} \} \) for 8PSK\(^1\)) with variance \( \sigma_e^2 \) are fed into the encoder, which generates transmit sequences \( s_1[k], s_2[k] \) according to the law

\[
\begin{align*}
    s_1[k] &= \begin{cases} 
        c_1[\mu], & k = 2\mu, \\
        -c_2[\mu], & k = 2\mu + 1, 
    \end{cases} \\
    s_2[k] &= \begin{cases} 
        c_2[\mu], & k = 2\mu, \\
        c_1[\mu], & k = 2\mu + 1 
    \end{cases}
\end{align*}
\]

((.)*: complex conjugation). Because of the repetition of signal points, the processes \( s_1[\cdot], s_2[\cdot] \) are jointly cyclostationary with period 2. Therefore, it is convenient to organize the polyphase components of \( s_1[\cdot], s_2[\cdot] \) in a vector

\[
    \begin{bmatrix}
        s_1[2\mu] \\
        s_2[2\mu] \\
        s_1[2\mu + 1] \\
        s_2[2\mu + 1]
    \end{bmatrix}
\]

\((\cdot)^T\): transposition), whose statistical properties are analyzed in the following. \( s[\cdot] \) is a stationary vector process with autocorrelation

\[
    \Phi_{ss}[\cdot] \triangleq E\{ s[\mu] s^H[\mu - \kappa] \} = \delta[\kappa].
\]

\[
    \begin{pmatrix}
        0 & 0 & 0 & \xi_e^2 \\
        0 & 0 & -\xi_e^2 & 0 \\
        0 & -\xi_e^2 & 0 & 0 \\
        \xi_e^2 & 0 & 0 & 0
    \end{pmatrix}
\]

where \( E\{ \cdot \}, (\cdot)^H, \delta[\cdot] \), and \( E_K \) denote expectation, Hermitian transposition, the unit pulse sequence, and the \( K \times K \) identity matrix, respectively. In addition, the definition

\[
    \xi_e^2 \triangleq E\{ |c_1[\mu]|^2 \} = E\{ |c_2[\mu]|^2 \}
\]

has been used in Eq. (4). E.g., \( \xi_e^2 = 1/2 \) for BPSK and \( \xi_e^2 = 0 \) for \( M \)-ary PSK with \( M > 2 \). For complex random processes, the pseudocorrelation [9]

\[
    \Phi_{ss*}[\cdot] \triangleq E\{ s[\mu] s^T[\mu - \kappa] \}
\]

is another important characteristic containing information about the statistical properties of the process. For vanishing pseudocorrelation, the process is referred to as proper (rotationally invariant) [9]. This holds for most complex random processes occurring in digital communications. Interestingly, \( \Phi_{ss*}[0] \neq 0 \) holds for the transmit vector of Alamouti’s scheme (and most other relevant space-time block coding schemes):

\(^1\)In general, a normalization factor of \( \frac{1}{\sqrt{P}} \) is included in the signal set in order to guarantee that the total transmit power is equal for space-time encoded and uncoded transmission.
\[ \Phi_{ss}[k] = \delta[k] \cdot \sigma_s^2 \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_1^2 * \xi_1^2 \end{bmatrix} \]

Hence, \( s[n] \) is an improper (rotationally variant) process \( [9] \) for any real or complex signal constellation \( \mathcal{C} \). For improper transmit signals and/or channel noise, performance of a linear receiver can be improved by widely linear (WL) processing \([12]\). In a WL receiver, the received sequence and its complex conjugate are processed jointly. In \([10]\), the concept of widely linear estimation has been applied to the equalization problem for transmission of real-valued data over single-input single-output (SISO) SI channels with complex-valued coefficients. In this paper, we utilize the concept of WL processing for low-complexity equalization of space-time block encoded transmission over multiple-input multiple-output (MIMO) channels. In particular, we propose a widely linear equalizer and a decision-feedback equalizer based on widely linear processing.

At the receiver, \( N_R \) antennas are employed. In this paper, we will pay special attention to the case \( N_R = 1 \), which is most relevant for space-time coding. The discrete-time received signal at antenna \( m \), \( m \in \{1, \ldots, N_R\} \), may be written as

\[ r_m[k] = \sum_{i=1}^{N_R} \sum_{n=0}^{\tilde{q}_m} h_{im}[n] s_i[k - n] + n_m[k], \]

where \( h_{im}[n] \) denotes the causal discrete-time impulse response of (maximal) order \( \tilde{q}_m \) characterizing transmission from transmit antenna \( i \) to receive antenna \( m \) including continuous-time transmit and receiver input filtering, \( n_m[n], m \in \{1, \ldots, N_R\} \), are mutually independent proper complex additive white Gaussian noise processes with equal powers, \( E\left[|n_m[k]|^2\right] = \sigma_n^2 \), \( \forall m \).

### 3 Widely Linear Equalization

In a direct approach to receiver design, equalization and space-time block decoding may be separated, cf. e.g. \([8]\). First, the matrix channel is equalized into a temporal ISI-free channel response. Assuming linear equalization, in this first stage an \( N_T \times N_R \) linear MIMO filter with transfer matrix \( \hat{F} \) is employed, whose coefficients are selected according to the minimum mean-squared error (MMSE) or the zero-forcing (ZF) criterion. Spatial ISI is retained in the equalizer output signal in order to achieve a diversity gain with the subsequent space-time block decoding \([8]\).

ZF equalization is possible if and only if

\[ \exists \bar{z} \in \mathbb{C} \setminus \{0, \infty\} \text{ with } \text{rank}(H(z)) = N_T, \]

where \( H(z) \) denotes the MIMO channel transfer matrix corresponding to the impulse responses \( h_{im}[n] \). Otherwise, the ZF condition

\[ F(z) H(z) = E_N \]

cannot be fulfilled. Eq. (9) is not valid for pure transmit diversity \( N_T > 1, N_R = 1 \), which is the most relevant scenario for space-time coding. Hence, ZF equalization is not possible in this case using the direct approach. On the other hand, also MMSE equalization usually yields a poor performance if a ZF equalizer does not exist, cf. e.g. \([13]\).

In our alternative approach, the space-time block encoder and the channel are viewed as an equivalent overall channel at the receiver side, for which a WL equalizer is designed for direct reconstruction of the sequences \( c_1[n], c_2[n] \). For this, the cyclostationarity of the received signal has to be taken into account. Time-invariant equalizer filters are obtained if a vector containing the polyphase components of each received sequence is taken as equalizer input \([14]\). Because our aim is to apply WL processing in order to utilize the rotational variance of the transmit signal, this vector is augmented by the complex-conjugates of the polyphase components. Hence, for derivation of the WL equalizer, we first define

\[ \tilde{r}_m[n] = \frac{1}{N_T} \sum_{\nu=0}^{N_T-1} \sum_{\mu=0}^{N_R-1} r_m[\nu + \mu, n] \]

Each sequence \( \tilde{r}_m[n] \) represents a stationary vector process, and

\[ \hat{r}_m[n] = \sum_{\nu=0}^{\tilde{q}_m} \sum_{\mu=0}^{\tilde{q}_m} \tilde{H}_m[\nu] \tilde{c}_m[\mu - \nu] + \tilde{n}_m[n] \]

is valid, with

\[ \tilde{c}_m[\mu] = [c_1[\mu], c_2[\mu], c_1^*[\mu], c_2^*[\mu]]^T \]

\[ \tilde{n}_m[\nu] = [n_m[\nu, 2\mu + 1], n_m[\nu, 2\mu + 1]^T \]

The coefficient matrices \( \tilde{H}_m[\nu] \) depend in a straightforward way on the channel coefficients \( h_{im}[n] \). Finally,

\[ \tilde{r}[n] = \tilde{r}_1[n] \ldots \tilde{r}_{N_R}[n] = \sum_{\nu=0}^{\tilde{q}_m} \tilde{H}[\nu] \tilde{c}[\nu - \nu] + \tilde{n}[\nu] \]

can be obtained, with obvious definitions of \( \tilde{H}[\nu] \) and \( \tilde{n}[\nu] \). The channel orders of equivalent overall and original channel are related by \( \tilde{q}_m = \lceil \frac{Q_m}{N_T} \rceil \) \([\lceil x \rceil: \text{smallest integer } \geq x]\). Eq. (15) represents a MIMO system with 4 inputs and 4 \( N_R \) outputs, with a \( 4 N_R \times 4 \) transfer matrix \( \tilde{H}(z) \). For all cases of practical interest,
the subchannel polyphase components are sufficiently different in order to guarantee that Eq. (9) is fulfilled for $H(z)$. Especially, for $N_R = 1$ an equalizable $4 \times 4$ MIMO system arises. Hence, for any $N_R \geq 1$ a transfer matrix $\tilde{F}(z)$ exists with

$$\tilde{F}(z) \quad H(z) = E_4.$$  \hspace{1cm} (16)

Thus, a WL ZF equalizer can be constructed.

Because ZF equalizers are available, it can be also expected that MMSE equalization performs well [13].

A WL MMSE equalizer with infinite order can be designed applying general results on equalizers for MIMO channels given in [14, 15], and also FIR WL MMSE equalizers may be derived.

An example, we consider widely linear ZF equalization for a flat fading matrix channel with $N_R = 1$. Here, $\tilde{H}(z)$ simplifies to

$$\tilde{H}(z) = \tilde{H}[0] = \begin{bmatrix} h_{11}[0] & h_{21}[0] & 0 & 0 \\ 0 & 0 & h_{21}[0] & -h_{11}[0] \\ h_{11}^*[0] & h_{11}^*[0] & h_{21}^*[0] & 0 \\ 0 & 0 & h_{21}^*[0] & 0 \end{bmatrix},$$  \hspace{1cm} (17)

Because $\tilde{H}[0]$ is a unitary matrix (up to a constant factor),

$$\tilde{H}^*[0] \tilde{H}[0] = (|h_{11}[0]|^2 + |h_{21}[0]|^2) \quad E_4,$$  \hspace{1cm} (18)

a linear ZF equalizer is given by

$$\tilde{F}(z) = \tilde{F}[0] = \frac{1}{|h_{11}[0]|^2 + |h_{21}[0]|^2} \tilde{H}^*[0],$$  \hspace{1cm} (19)

producing an output sequence $\tilde{y}[\mu] = \tilde{F}[0] \tilde{r}[\mu] = [c_1[\mu] \quad c_2[\mu] \quad c_1^*[\mu] \quad c_2^*[\mu]]^T + n_{\tilde{F}}[\mu]$, where the autocorrelation of the output noise $n_{\tilde{F}}[\mu]$ is $E(\tilde{n}_{\tilde{F}}[\mu] n_{\tilde{F}}^*[\mu - \nu]) = 1/(|h_{11}[0]|^2 + |h_{21}[0]|^2) \delta[\nu] E_4$.

Thus, the first and second entry of the equalizer output vector $\tilde{y}[\mu]$, $\tilde{y}_1[\mu] \triangleq 1/(|h_{11}[0]|^2 + |h_{21}[0]|^2) (h_{11}[0] r_1[2 \mu] + h_{21}[0] r_1^*[2 \mu + 1])$, $\tilde{y}_2[\mu] \triangleq 1/(|h_{11}[0]|^2 + |h_{21}[0]|^2) (h_{11}[0] r_1^*[2 \mu] - h_{11}[0] r_1^*[2 \mu + 1])$ are sufficient for estimation of $c_1[\mu], c_2[\mu]$; applying quantization to the nearest signal points. In fact, this procedure is equivalent to maximum-likelihood (ML) detection of space-time block-encoded signals transmitted over flat fading channels. The described strategy is well known as the standard detection rule for STBC’s [3, 16]. It represents a simplified version of straightforward ML detection. According to our derivation, the rule arises naturally in a WL processing context. It should be noted that only in the flat fading case the sequences $r_1^*[2 \mu]$ and $r_1[2 \mu + 1]$ are not required for detection, because only here the corresponding entries of the ZF equalizer matrix are zero, cf. Eqs. (19) and (17). Otherwise, WL equalization has to consider each polyphase component as well as its complex conjugate.

### 4 Widely Linear Decision Feedback Equalization

It is well known that performance of a linear equalizer can be improved by employing noise prediction or, equivalently, decision feedback. A MIMO decision-feedback equalizer (DFE) consists of a feedforward filter $\tilde{F}(z)$ and a feedback filter $\tilde{B}(z) = E_4$, where $\tilde{B}(z)$ is a causal and monic matrix polynomial,

$$\tilde{B}(z) = \sum_{\nu=0}^{N_R} \tilde{n}_B[\nu] z^{-\nu}, \quad \tilde{B}[0] = E_4.$$  \hspace{1cm} (20)

Fig. 1 shows the structure of a WL DFE for Alamouti’s scheme. It should be noted that estimates for $c_1[\mu]$ and $c_2[\mu]$ can be obtained by conjugating $c_1[\mu]$ and $c_2[\mu]$. This explains the open lines in Fig. 1. As a consequence, only half of the scalar filters in matrices $\tilde{F}(z)$ and $\tilde{B}(z) = E_4$ need to be implemented in practice. For the special case $\tilde{B}(z) = E_4$, a widely linear equalizer according to Section 3 results.

![Figure 1: Decision-feedback equalizer with widely linear processing for space-time block-encoded transmission.](image)

Figure 1: Decision-feedback equalizer with widely linear processing for space-time block-encoded transmission. $\downarrow 2$: Downsampling by factor 2.

The optimum infinite-length filters for a MIMO--DFE have been derived, e.g., in [14] for square MIMO systems. Hence, these results can be used for the design of a WL decision-feedback equalizer with $N_R = 1^2$. For arbitrary nonsquare MIMO systems, the infinite-length MMSE-DFE is derived in [15]. Also, FIR MMSE-DFE receivers with WL processing may be derived, which are primarily of interest for practical applications.

### 5 Results for GSM/EDGE

In the following, the proposed receiver concepts are applied to an EGPRS (Enhanced General Packet Radio Service) transmission system, which is part of

2The results of [14] can be also used for derivation of optimum ZF--DFE filters, if it is taken into account that the MMSE solution for i.i.d. data becomes identical to the ZF solution if the signal-to-noise ratio tends to infinity.
the EDGE standard. Nine modulation and coding schemes (MCSs) are specified for EGPRS. Four schemes (MCS-1-MCS-4) employ binary Gaussian minimum-shift keying (GMSK) modulation, which is also applied in GSM and can be well approximated by filtered BPSK modulation. For good channel conditions, a higher throughput can be achieved with MCS-5-MCS-9, which are based on 8PSK modulation. For GSM/EDGE four different power delay profiles are specified [17], rural area (RA), hilly terrain (HT), typical urban area (TU), and equalizer test (EQ). For each transmit and receive branch, a GMSK pulse and a square-root raised cosine filter with roll-off factor \( \alpha = 0.3 \), respectively, has been selected for continuous-time filtering, cf. also [18]. Unless otherwise stated, the availability of ideal channel estimates at the receiver side is assumed, and only a single receive antenna has been used (\( N_R = 1 \)).

In Fig. 2, the bit error rate (BER) versus \( 10 \log_{10}(E_b/N_0) \) (\( E_b \): average received bit energy per receive antenna, \( N_0 \): noise power spectral density) of (widely) linear equalization for conventional and space-time encoded transmission is shown for the RA profile and 8PSK transmission. For the STBC case, a WL FIR MMSE equalizer of order 10 has been designed. Also, curves for real channel estimation are given. Fig. 2 shows that the loss due to channel estimation is larger for space-time encoded transmission than for SISO transmission. This is because in the former case a larger number of channel coefficients has to be estimated, and the energy of the training sequences is reduced due to multiple transmit antennas. Nevertheless, also for nonideal channel estimation STBC-MMSE-WLE clearly outperforms SISO-MMSE-LE.

Next, we study the performance of BPSK transmission over an HT channel assuming a DFE in the receiver. Fig. 3 indicates a gain of \( 8 \pm 2 \) dB at BER = 10\(^{-3} \) for STBC-MMSE-WDFE compared to SISO-MMSE-DFE.

![Figure 3: BER vs. 10log10(E_b/N_0) for BPSK transmission over HT channel and (W)DFE (NR = 1).](image)

Finally, according to Fig. 4, which is valid for TU, 8PSK transmission, DFE equalization, and \( N_R = 2 \), WL equalization using the structural properties of the STBC is also beneficial if additional receive diversity is available.

![Figure 4: BER vs. 10log10(E_b/N_0) for 8PSK transmission over TU channel and (W)DFE (additional receive diversity, NR = 2).](image)
Conclusions

In this paper, we consider a space–time block–encoded transmission over frequency–selective fading channels. Utilizing the cyclostationarity and the rotational variance of the transmitted signal, we propose several widely linear detectors, which also process the complex-conjugated versions of the received polyphase components. With this approach, high performance of detection can be also guaranteed for pure transmit diversity, in contrast to previously proposed schemes. Simulation results for EDGE show that space–time block–encoded transmission combined with a WL receiver outperforms a SISO transmission combined with the conventional counterpart of the corresponding WL receiver, if inherent temporal channel diversity is small to moderate.

References


