A Novel Metric for Noncoherent Sequence Estimation

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Abstract

In this letter, a novel simple noncoherent sequence estimation (NSE) metric for M-ary differential phase-shift keying (MDPSK) transmission over intersymbol interference (ISI) channels is proposed. In contrast to previously reported NSE schemes (cf. [1] and references therein), the phase reference symbol required for metric calculation can be obtained efficiently in a recursive manner. Simulations demonstrate the high performance of the novel scheme which can approach the performance of coherent maximum-likelihood sequence estimation (MLSE) while complexity of metric computation remains relatively low.

Index Terms: Digital communication, noncoherent sequence estimation, equalization.

1 Introduction

Recently, various noncoherent sequence estimation (NSE) schemes for M-ary differential phase-shift keying (MDPSK) transmission over intersymbol interference (ISI) channels have been proposed (cf. [1] and references therein). Since the optimum NSE metric cannot be implemented efficiently, approximations are necessary. Here, the most promising technique reported so far is the so-called Forney approach of Colavolpe and Raheli [1]. The novel scheme proposed in this letter offers the same power efficiency like the scheme of [1], however, metric calculation is less complex.

2 Transmission Model

In this letter, all signals are represented by their complex-valued baseband equivalents. At the transmitter, the MDPSK symbols $a[\cdot] \in \mathcal{A} = \{e^{j2\pi \nu/M} | \nu \in \{0, 1, \ldots, M-1\}\}$ are differentially encoded. The resulting MPSK symbols $b[\cdot] \in \mathcal{A}$ are given by $b[k] = a[k]b[k-1]$, $k \in \mathbb{Z}$. The discrete-time received signal, sampled at times $kT$ ($T$ is the symbol interval) at the output of the
receiver input filter, can be written as
\[ r[k] = e^{j\theta} e^{j2\pi\Delta f T k} \sum_{\nu=0}^{L-1} h_{\nu} b[k - \nu] + n[k]. \tag{1} \]

Here, \( h_{\nu}, 0 \leq \nu \leq L - 1 \), denote the coefficients of the combined discrete-time impulse response of transmit filter, channel, and receiver input filter. \( L \) is the length of the impulse response. \( \theta \) denotes an unknown, constant, uniformly distributed phase shift. The effects of an uncompensated phase drift caused by a demodulator frequency offset \( \Delta f \) are modeled by a multiplicative factor \( e^{j2\pi\Delta f T k} \). We assume a square-root Nyquist frequency response for the receiver input filter, and thus, the zero mean complex Gaussian noise \( n[\cdot] \) is white. Due to an appropriate normalization, the variance of \( n[\cdot] \) is \( \sigma_n^2 = \mathcal{E}\{|n[k]|^2\} = N_0/E_S \), where \( \mathcal{E}\{\cdot\} \) denotes expectation. \( E_S \) and \( N_0 \) are the mean received energy per symbol and the single-sided power spectral density of the underlying passband noise process, respectively.

### 3 Novel Metric

For derivation of the novel metric, we assume zero frequency offset (\( \Delta f = 0 \)), however, in the simulations presented in Section 4, the influence of frequency offset on the receiver performance is also investigated.

At high signal-to-noise ratios, the optimum NSE metric for estimation of a block of \( N_T - 1 \) transmitted symbols \( a[k], 1 \leq k \leq N_T - 1 \), is well approximated by [1]
\[ \Lambda[N_T] \triangleq -\frac{1}{2} \sum_{\nu=0}^{N_T-1} |\tilde{y}[\nu]|^2 + \left| \sum_{\nu=0}^{N_T-1} r[\nu] \tilde{y}^*[\nu] \right|, \tag{2} \]

where the definition \( \tilde{y}[k] \triangleq \sum_{\mu=0}^{L-1} h_{\mu} \tilde{b}[k - \mu] \) is used and \((\cdot)^*\) denotes complex conjugation. Here, \( \tilde{b}[\cdot] \in \mathcal{A} \) are hypothetical equalizer trial symbols. In practice, the impulse response coefficients \( h_{\mu}, 0 \leq \mu \leq L - 1 \), have to be estimated using an appropriate noncoherent channel estimation algorithm [2]. For simplicity, in this letter perfect knowledge of the channel impulse response is assumed.

From Eq. (2), it can be concluded that the incremental metric \( \lambda[k] = \Lambda[k+1] - \Lambda[k] \) at time \( k, 0 < k \leq N_T - 1 \), is given by [1]
\[ \lambda[k] = |r[k] \tilde{y}^*[k] + \tilde{a}_{\text{ref}}[k-1]| - |\tilde{a}_{\text{ref}}[k-1]| - \frac{1}{2} |\tilde{y}[k]|^2, \tag{3} \]

with the definition \( \tilde{a}_{\text{ref}}[k-1] \triangleq \sum_{\nu=0}^{k-1} r[\nu] \tilde{y}^*[\nu] \). \( \tilde{a}_{\text{ref}}[k-1] \) can be considered as phase reference symbol. In its present form \( \tilde{a}_{\text{ref}}[k-1] \) corresponds to an unlimited phase memory which grows with time \( k \). Thus, it is not possible to use the Viterbi algorithm for maximization of \( \Lambda[N_T] \). An even more important disadvantage is that a severe performance degradation occurs if the channel phase
varies due to frequency offset or fading. In order to resolve these problems Colavolpe and Raheli [1] use a rectangular window to limit the phase memory, i.e., \( \tilde{q}_{\text{ref}}^N[k-1] \) in Eq. (3) is approximated by \( \tilde{q}_{\text{ref}}^N[k-1] = \sum_{\nu=1}^{N-1} r[k - \nu] \tilde{y}^*[k - \nu] \). \( N - 1, N \geq 2 \), is the window size.

For the novel metric, we propose to use an exponential window instead of the rectangular window. This has the important advantage that the modified reference symbol \( \tilde{q}_{\text{ref}}^\alpha[k-1] \) can be generated recursively, i.e., \( \tilde{q}_{\text{ref}}^\alpha[k-1] = \alpha \tilde{q}_{\text{ref}}^\alpha[k-2] + r[k-1] \tilde{y}^*[k-1] \). \( \alpha, 0 \leq \alpha < 1 \), is a forgetting factor. Note, that for the special cases \( N = 2 \) \( (N \to \infty) \) and \( \alpha = 0 \) \( (\alpha \to 1) \) \( \tilde{q}_{\text{ref}}^N[k-1] \) and \( \tilde{q}_{\text{ref}}^\alpha[k-1] \) are identical.

In order to further limit complexity and for a fair comparison with coherent maximum-likelihood sequence estimation (MLSE), we employ per-survivor processing [3] and define a trellis diagram with \( M^K \) states \( S[k] \triangleq (\hat{b}[k-1], \hat{b}[k-2], \ldots, \hat{b}[k-K]) \), \( 0 \leq K \leq L - 1 \). This measure enables the application of the Viterbi algorithm to the novel scheme. The corresponding branch metrics are given by

\[
\lambda^\alpha[k] = |r[k] \tilde{y}^*[k] + \tilde{q}_{\text{ref}}^\alpha(k-1, S[k])| - |\tilde{q}_{\text{ref}}^\alpha(k-1, S[k])| - \frac{1}{2} |\tilde{y}[k]|^2,
\]

with

\[
\tilde{q}_{\text{ref}}^\alpha(k-1, S[k]) = \alpha \tilde{q}_{\text{ref}}^\alpha(k-2, S[k-1]) + r[k-1] \tilde{y}^*[k-1].
\]

The new notation in Eq. (5) indicates that each state \( S[k] \) has an associated reference symbol \( \tilde{q}_{\text{ref}}^\alpha(k-1, S[k]) \). For \( 0 \leq \nu \leq K \), the hypothetical symbols \( \tilde{b}[k-\nu] \) in \( \lambda^\alpha[k] \) are defined by the transition \( t[k] \triangleq (\tilde{b}[k], \tilde{b}[k-1], \ldots, \tilde{b}[k-K]) \) from state \( S[k] \) to \( S[k+1] \). For \( \nu > K \), the symbols \( \tilde{b}[k-\nu] \) are taken from the surviving path terminating in state \( S[k] \). The reference symbol \( \tilde{q}_{\text{ref}}^\alpha(k-2, S[k-1]) \) in Eq. (5) is taken from that state \( S[k-1] \) which is contained in the surviving path associated with \( S[k] \).

As typical for coherent MLSE, we use a decision delay of \( k_0 = 5K \) symbols, i.e., at time \( k \) the estimated MPSK symbol \( \hat{b}[k-k_0] \) is taken from the surviving path with maximum accumulated metric. Then, the estimated MDPSK symbol \( \hat{a}[k-k_0] \) is obtained by inverting differential encoding \( (\hat{a}[k-k_0] = \hat{b}[k-k_0] \hat{b}^*[k-k_0-1]) \).

A comparison of Eqs. (4), (5) with the corresponding equations for metric calculation in [1] shows that for the proposed scheme less arithmetic operations are necessary. In particular, complexity is independent of \( \alpha \) for the proposed scheme but increases with \( N \) for the scheme of [1]. Thus, the savings in computational complexity offered by the novel scheme become the larger the larger \( N \) is chosen, while the same power efficiency as for the scheme of [1] can be always obtained by choosing \( \alpha \) properly.
4 Simulation Results

In this section, we present some simulation results for the proposed NSE scheme and QDPSK ($M = 4$) transmission. First a channel with impulse response $h_0 = 0.304$, $h_1 = 0.903$, $h_2 = 0.304$ and $L = 3$ is chosen. Fig. 1 shows the bit error rate (BER) vs. $10 \log_{10}(E_b/N_0)$ ($E_b = E_s/2$ is the mean received energy per bit) for the proposed NSE scheme employing different forgetting factors $\alpha$. For NSE, $K = 2$ is valid, i.e., the same number of states is used as required for coherent MLSE employing the standard Viterbi algorithm. For comparison the BER of coherent MLSE is also included in Fig. 1. It can be observed that the novel NSE scheme approaches coherent MLSE as $\alpha$ approaches unity. Note, that the NSE scheme proposed in [1] also approaches coherent MLSE for $N \to \infty$.

Fig. 2 shows BER vs. normalized frequency offset $\Delta fT$ for the novel NSE scheme (solid lines). Here, the channel impulse response is $h_0 = 0.407$, $h_1 = 0.815$, $h_2 = 0.407$ ($L = 3$). Furthermore, $K = 2$ and $10 \log_{10}(E_b/N_0) = 11$ dB are valid. For comparison, the BER curves for the original NSE scheme (Forney Approach) of [1] employing the same number of states are also included for various window sizes $N$ (dashed lines). Note, that for $N = 2$ and $\alpha = 0$ both schemes are identical. Clearly, there is a trade-off between power efficiency for $\Delta f = 0$ and robustness against frequency offset. As $\alpha (N)$ increases, the robustness against phase variations decreases, while power efficiency for $\Delta f = 0$ increases. In a practical application, $\alpha (N)$ has to be optimized depending on the maximum expected frequency offset. For the novel scheme this optimization is facilitated since $\alpha$ is a real number. On the other hand, $N$ is integer and the optimum point, i.e., the desired BER for given $E_b/N_0$ ratio and frequency offset $\Delta f$ cannot be always attained exactly.

5 Conclusions

In this letter, a novel simple NSE metric is proposed. In contrast to previously reported schemes, an exponential window is used to limit the memory of the phase reference symbol necessary for metric calculation. Thus, here the reference symbol can be generated recursively. The number of arithmetic operations required for metric computation is independent of the forgetting factor $\alpha$, whereas for the original scheme it increases linearly with the window size $N$. Although, in this letter, we restricted ourselves to uncoded transmission over ISI channels, the novel metric might be applied successfully to any noncoherent sequence estimation problem, e.g. noncoherent sequence estimation for convolutionally encoded MDPSK transmission or trellis-coded modulation.
References


Figure 1: BER vs. $10 \log_{10}(E_b/N_0)$ for the proposed NSE scheme and coherent MLSE ($h_0 = h_2 = 0.304, h_1 = 0.903$, QDPSK transmission).

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novel NSE scheme

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coherent MLSE


\[ \nabla : \alpha = 0, \quad \square : \alpha = 0.2, \quad \times : \alpha = 0.4, \quad \circ : \alpha = 0.6, \quad \triangle : \alpha = 0.8, \quad \ast : \alpha = 0.9 \]
Figure 2: BER vs. $\Delta f T$ for the novel NSE scheme (solid lines) and the NSE scheme proposed in [1] (dashed lines) ($h_0 = h_2 = 0.407$, $h_1 = 0.815$, $10 \log_{10}(E_b/N_0) = 11$ dB, QDPSK transmission).

$\nabla \nabla : \alpha = 0$, $\square \square : \alpha = 0.2$, $\times \times : \alpha = 0.4$, $\circ \circ : \alpha = 0.6$, $\triangle \triangle : \alpha = 0.8$, $\leftrightarrow : \alpha = 0.9$,

$\square - \square : N = 3$, $\circ - \circ : N = 5$, $\triangle - \triangle : N = 10$