Abstract — The capacity of flat fading channels when applying differentially encoding with non-coherent reception and no channel state information available at the receiver is considered. Numerical results indicate the gains achievable by multiple symbol detection in the case of slowly time-varying channels and provide a comparison among schemes with different potential bandwidth efficiencies.

1 Introduction

In many situations, for transmission over flat fading channels, neither channel state information is available nor coherent reception is possible. For such applications it is convenient to employ differential encoding, i.e., to represent information in the transitions between consecutive symbols rather than in the symbols themselves (cf. e.g. [1]). For high bandwidth efficiencies this is done with respect to phase, as well as with respect to amplitude of APSK constellations. If the underlying channel is slowly time-varying, interleaving and detection can be based on blocks of $N$ consecutive symbols. In this paper a vector channel model describing the transmission scheme is established. Based on this channel model, capacity curves for several signal constellations and differentially encoded transmission with multiple symbol detection are presented. The special case of pure phase modulation over the additive white Gaussian noise channel with unknown carrier phase was regarded in [2].
2 System Model and Capacity

The transmission scheme applied throughout the letter is sketched in Figure 1. Core is the actual, discrete-time channel model, given in the equivalent low-pass domain [3]. The channel is assumed to be a stationary, slowly time-varying, frequency non-selective (non-dispersive) Rician fading channel with finite memory. As usual, channel state and carrier phase offset are expected to be constant over a block of $N$ consecutive symbols.

![Figure 1: System model.](image)

In order not to require knowledge on the carrier phase and actual channel state, we employ differential encoding at the transmitter and differential detection at the receiver. As in this letter the coherence time of the channel is (at least) $N$ symbols, the encoder operates on blocks $x$ of $N$ consecutive channel input symbols $x$. Given one symbol $s$ of the preceding block as reference, $N - 1$ transitions are determined by the vector $a$ consisting of $N - 1$ differential symbols $a$.  

Favorable signal constellations $X$ for the transmit symbols $x$ consist of $\alpha \cdot \beta$ points arranged in $\alpha$ distinct concentric rings with different radii $r_i$, $i = 0, 1, \ldots, \alpha - 1$, and $\beta$ uniformly spaced phases. We denote these constellations, also known as star QAM, by $\alpha A\beta$PSK. For convenience, we restrict the differential symbols $a$ to be taken from the same signal set as the transmit signal. Then, a simple formulation of the operation of the differential encoder results. Given a reference symbol $s = r_i e^{j \varphi_n}$ and a differential symbol $a = r_j e^{j \varphi_m}$ (phase and amplitude increment), the encoder returns $x = r_{(i+j) \mod \alpha} e^{j (\varphi_n + \varphi_m)}$.

At the receiver, $N$ consecutive channel output symbols $y$ are grouped and comprised into the vector $y$. Thereby, the blocks are overlapping by one symbol [1]. For reasons of complexity it is common (cf. e.g. [1]) to ignore the statistical dependencies between blocks $y$ by performing ideal interleaving $I$ at the transmitter and the inverse operation ($I^{-1}$) at the receiver side.

After interleaving, the transmission between $a$ and $y$ — we denote it by vector channel — is regarded to be memoryless, and hence, can completely be characterized

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3For the moment, interleaving and deinterleaving are ignored.
by a single probability density function (pdf) $p_Y(y|a)$ of $y$ for given $a$. In order to obtain $p_Y(y|a)$ we consider the transmission of the $N$-dimensional symbols $\mathbf{a} := [s, \mathbf{x}]$ through the memoryless channel, modeled to be stationary, non-dispersive and multiplicative Rician fading. The corresponding pdf $p_Y(y|\mathbf{a})$, and hence $p_Y(y|a, s)$, is derived in the Appendix of [1]. Averaging $p_Y(y|a, s)$ with respect to $s$ yields $p_Y(y|a)$.

Having established the channel model the capacity of the memoryless vector channel can be calculated. The normalized capacity $C(N)$, measured in bit per symbol, is obtained by\(^4\) [4]

$$C(N) := \frac{1}{N-1} \cdot \mathbb{E}_{Y,a} \left\{ \log_2 \frac{p_Y(y|a)}{p_{\mathbf{X}}(y)} \right\}$$

where $\mathbb{E}\{\cdot\}$ denotes expectation and $p_Y(y)$ is the average pdf of the channel output. Regardless the choice of the distribution of $a$ the differentially encoded symbols $x$ are uniformly distributed over $\mathcal{X}$. If, furthermore, $x$ is uniformly, independently, and identically distributed (u.i.i.d) the entropy of the channel input is maximized. Because this implies an u.i.i.d. input to the differential encoder we choose the vector symbols $a$ to be u.i.i.d., and thus no optimization on the channel input distribution can be performed.

As already observed in [5] the normalized capacity increases with the block size $N$. If $N$ approaches infinity $C(N)$ converges to the capacity $C_{\text{CSI}}$ for the case of perfect channel state information (CSI) and coherent reception, which upper bounds $C(N)$.

3 Numerical Results and Discussion

Numerical results for the capacity $C(N)$ over the average signal-to-noise ratio $E_s/N_0$ are presented for the case of Rayleigh fading.

The influence of the block size $N$ on the normalized capacity $C(N)$ for 8-ary differential PSK (8DPSK) is illustrated in Figure 2. As expected, by enlarging $N$ the capacity $C(N)$ increases. But the convergency of $C(N)$ to $C_{\text{CSI}}$ as $N$ goes to infinity is rather slow. At $C = 1.5$ bit/ch. use the gap is approx. 3 dB for $N = 2$ and 2 dB for $N = 3$.

For D2APSK, the effect of increasing the block size $N$ on the capacity is shown in Figure 3. Here, the ring ratio $r_1/r_0 = 2.0$ was chosen [6]. Like above, the convergency

\(^4\)We denote random variables corresponding to signals by the respective capital letter.
of the normalized capacity to the capacity $C_{\text{CSI}}$ can be observed. Here, for $C = 2.5$ bit/ch. use a gap of approx. 4 dB for $N = 2$ and 3 dB for $N = 3$ results.

Finally, the capacities of various $\alpha A/3$PSK constellations and differentially encoded transmission over the Rayleigh fading channel are summarized in Figure 4. Again, for $\alpha = 2$ (D2A8PSK, D2A16PSK) the ring ratio equals 2.0. For $\alpha = 4$ (D4A16PSK) the rings are geometrically spaced with the ratio $r_{i+1}/r_i = 1.4$, $i = 0, 1, 2$. Note, optimally the ring ratio has to be optimized for each signal-to-noise ratio. Because of the fixed ratio, the capacity curves for DPSK and DAPSK intersect, and at low rates DAPSK is inferior. This effect is mitigated as the ring ratio goes to one.

It is interesting to observe that in contrast to coherent transmission over Rayleigh fading channels here channel symbol expansion diversity [7] is not expected to be favorable for low rates. Using large DAPSK constellations in combination with a low rate code does not promise better performance in terms of capacity than e.g. 4DPSK with code rate 1/2. On the other hand, however, for high spectral efficiencies some gains can be achieved by spending more than one bit of redundancy per channel use.

To summarize, the presented capacity curves constitute guidelines for the system design using differential encoding and powerful channel encoding. Depending on the desired power and bandwidth efficiency the appropriate modulation scheme and demodulation procedure can be assigned.
References


**Figure 2:** Normalized capacity for 8DPSK and $N = 2, 3, 4, 5$ (from right to left). Rayleigh fading channel. Dashed Line: Capacity $C_{\text{CSI}}$. 
Figure 3: Normalized capacity for D2A8PSK, with ring ratio $r_1/r_0 = 2.0$, and $N = 2, 3, 4$ (from right to left). Rayleigh fading channel. Dashed Line: Capacity $C_{\text{CSI}}$.

Figure 4: Capacities for 4DPSK, 8DPSK, D2A8PSK, D2A16PSK, D4A16PSK and $N = 2$ (from bottom to top). Rayleigh fading channel.