Widely-Linear Selected Mapping for Peak-to-Average Power Ratio Reduction in OFDM

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Abstract

Peak-to-average power ratio reduction in OFDM using selected mapping (SLM) is considered. A new low-complexity version of SLM is introduced, which employs the principle of widely-linear signal processing, i.e., real and imaginary part of the candidate OFDM frames are treated separately. Thereby, investing the numerical complexity of conventional SLM with \( U \) candidates, a performance almost equal to \( U^2 \) candidates can be obtained with widely-linear SLM. Numerical simulations cover this performance.

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Introduction — Orthogonal frequency-division multiplexing (OFDM), a particular implementation of multichannel modulation, is very popular for transmission over frequency-selective channels [2].

In an OFDM transmitter, a block of \(D\) distinct complex-valued carriers \(A_\mu, \mu = 0, \ldots, D - 1\), (OFDM frame) is transformed into time-domain (equivalent complex-valued baseband signals) using the inverse discrete Fourier transform (IDFT), i.e., \(a_k = \frac{1}{\sqrt{D}} \sum_{\mu=0}^{D-1} A_\mu \cdot e^{j2\pi k \mu / D}, k = 0, \ldots, D - 1\). The frequency-domain samples \(A_\mu\) are expected to be drawn from some (not necessarily the same) constellation with variance \(\sigma_a^2\). After introducing a “guard interval” [2], pulse shaping (including digital-to-analog conversion), and modulation to radio frequency, the signal is radiated from the antenna.

Unfortunately, because of the statistical independence of the carriers, the time-domain samples \(a_k\) are approximately Gaussian distributed [1]. This results in a high peak-to-average power ratio (PAR)\(^3\)

\[
\text{PAR} \triangleq \max_{k=0, \ldots, D-1} \frac{|a_k|^2}{\sigma_a^2},
\]

which causes non-linear distortion in the power amplifiers and in turn leads to the generation of undesired out-of-band radiation. In literature, a large number of PAR reduction techniques for OFDM are known. Thereby, selected mapping (SLM) [1], [7], [3] is of particular interest.

Review of Selected Mapping — Main idea of selected mapping is that each OFDM frame is mapped to a number of \(U\) (independent) candidates representing the same information. From these that one with the lowest PAR (or any other criteria) is selected.

In [1], [7] the candidates \(A^{(u)}\) are generated by multiplying carrier-wise the initial OFDM frame \(A = [A_0, \ldots, A_{D-1}]\) with \(U\) phase vectors \(P^{(u)} = [P_0^{(u)}, \ldots, P_{D-1}^{(u)}], u = 1, \ldots, U, P_\nu^{(u)} = e^{j \varphi^{(u)}}, \) and \(\varphi^{(u)}\) randomly selected (preferably drawn from \(\{0, \pi/2, \pi, 3\pi/2\}\)) and known to transmitter and receiver.

An alternative has been presented in [3]. Here, the candidates are generated by scrambling the block of binary data (filtering over the binary field with a pure recursive system) which is prefixed by \(\lceil \log_2(U) \rceil\) bits (\(\lceil x \rceil\): smallest integer \(\geq x\)) to represent \(U\) different labels (hence starting filtering of data with different states). After mapping to signal points, the candidates \(A^{(u)}\) are obtained.

In each variant of SLM, the candidates are transformed to time-domain \(a^{(u)} = \text{IDFT}\{A^{(u)}\}\) (\(U\) IDFTs are required) and the “best” OFDM frame \(a^{(u^*)}\) is actually transmitted, see top of Fig. 1. In order to recover data, in the first variant \(\lceil \log_2(U) \rceil\) bits of side information to indicate the vector \(P^{(u^*)}\) have to be communicated explicitly to the receiver. The advantage of the second variant is that no explicit side information has to be made aware to the receiver as it is implicitly included in the scrambled version; simple descrambling

\(^3\)Here we concentrate on the PAR of the discrete-time samples \(a_\mu\). An more accurate estimate for the PAR of the continuous-time transmit signal may be obtained by oversampling.
recovers data. However, in both cases, $\lceil \log_2(U) \rceil$ bits redundancy (either explicit phase vector number or scrambled, hence implicit, prefix label) have to be paid for PAR reduction.

Using SLM and assuming Gaussian time-domain samples, the probability that the PAR of an OFDM frame exceeds a certain threshold $\text{PAR}_0$ is given by \cite{1}

$$\Pr\{\text{PAR} > \text{PAR}_0\} = (1 - (1 - e^{-\text{PAR}_0})^{D_1})^U,$$ (2)

which shows that performance improves as $U$ increases.

**Widely-Linear Selected Mapping** — In \cite{6} a modified SLM scheme with lower complexity is proposed. Main idea is to suitably choose the phase vectors, such that combinations of pairs of candidate time-domain signals can also be used for transmission. As a result, using only $U$ IDFTs, the performance SLM which uses almost $U^2$ IDFTs can be achieved.

A more simple and obvious procedure than that in \cite{6} is as follows: In a number of applications, the use of so-called “widely-linear” signal processing is rewarding. Thereby, real and imaginary part of the equivalent complex baseband signal are separated and individually treated. Since the operation on pairs of real signals is more general than operations with complex numbers, more degrees of freedom are present and, hence, performance gains can be achieved, cf. \cite{8}, \cite{4}.

The idea of widely-linear processing can also be applied to PAR reduction. In Fig. 1 (bottom), the block diagram of the modified phase vector variant of SLM is shown. Now, for both, real ($A_I \overset{\text{def}}{=} \text{Re}\{A\}$) and imaginary part ($A_Q \overset{\text{def}}{=} \text{Im}\{A\}$) of the frequency-domain vectors, $U$ candidates are generated. Since the signals are real-valued, the phase vectors $P^{(u)}_{I/Q}$ (not necessarily the same for both quadrature components) have to be real, too, i.e., $P^{(u)}_{I/Q,d} \in \{-1, +1\}$. These candidates are transformed into time-domain using $2U$ “real-to-complex” IDFTs. Since $A^{(u)}_{I/Q}$ are real, the corresponding time-domain signals $a^{(u)}_{I/Q}$ exhibit hermitian symmetry, hence, only half of the time-domain samples have actually to be calculated. In turn, the complexity of $2U$ “real-to-complex” IDFTs is (almost) the same as $U$ complex ones (cf. also the discussion in \cite{6}).

In the last step, each combination of one real and one imaginary candidate is formed, leading to $U^2$ candidate time-domain signals. From these, the best candidate $a^{(u^*)} = a^{(u^*_I)} + j a^{(u^*_Q)}$ is selected. Consequently, performance of conventional SLM with $U^2$ candidates ($U$ in (2) replaced by $U^2$) is possible.

Noteworthy, this procedure also applies to the scrambler variant of SLM (independent scramblers and labels for real and imaginary part are used) and any generalisations of SLM, e.g., that to multi-antenna transmission \cite{5}. In each case, the performance improvement is paid with a double number of redundant bits (side information) but not complexity (except additional PAR calculations).
**Numerical Results and Conclusion** — To assess the performance of the proposed SLM scheme, numerical simulations have been conducted. The number of carriers (all used) is \( D = 512 \), and the modulation in each carrier is 4PSK. \( U = 4 \) or 16 phase vectors with phases randomly selected from the set \( \{0, \pi/2, \pi, 3\pi/2\} \) (conventional SLM, pure inversion or interchange of the quadrature components [1]), or \( \{0, \pi\} \) (“widely-linear” SLM, wLSLM) are used.

In Fig. 2, the complementary cumulative distribution function (ccdf) \( \Pr\{\text{PAR} > \text{PAR}_0\} \) is plotted for the variants of SLM. For reference (rightmost, dotted curve), the ccdf of OFDM without PAR reduction is included. It is clearly visible that wLSLM shows much better performance than conventional SLM employing the same complexity. wLSLM with \( U = 2 \) (4/8/16) candidates (per quadrature component) performs almost the same as ordinary SLM with \( 4 \) (16/64/256) candidates. The theoretical curves (gray) from (2) support this observation. For clipping levels in the order of \( 10^{-5} \) gains over 1 dB are possible due to the widely-linear approach.

Finally, please note that additional simulations cover that exactly the same performance for conventional and widely-linear version as in Fig. 2 is achieved, if the scrambler version of SLM [3] is used.

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**REFERENCES**


Fig. 1. Variants of selected mapping (version with phase vectors). Top: conventional (complex-valued) SLM with $U$ candidates/IDFTs; Bottom: widely-linear (real-valued) SLM (wlSLM) with the same complexity ($2U$ “real-to-complex” IDFTs). The best combination of real (I) and imaginary (Q) component is selected.
Fig. 2. Complementary cumulative distribution function (ccdf) $\Pr\{\text{PAR} > \text{PAR}_0\}$ for OFDM with conventional and widely-linear SLM. $D = 512$ carriers, $U = 2, 4, 8$, and 16 candidate vectors. Rightmost curve (dotted): MIMO OFDM without PAR reduction; gray: theoretical curves assuming Gaussian time-domain samples; (2) with (right to left) $U = 1, 2, 4, 8, 16, 64, 256$. 