Optimizing MIMO DFE for Systems with Spatial Loading

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***Abstract*—In this paper we consider transmission over MIMO channels with channel state information available at the transmitter. This enables an optimization of the rate and power distribution over the parallel subchannels in the transmission system (loading). We show that spatial loading provides substantial gains and give an algorithm to calculate the matrix filters required for decision feedback equalization (DFE) in systems with loading.**

Introduction — We study the transmission over MIMO channels, described in the discrete time equivalent complex baseband model by $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$. For simplicity we assume an equal number of K transmit and receive antennas. The matrix $\boldsymbol{H} = \left[h_{kl}\right]$ comprises the complex fading coefficients between transmit antenna l and receive antenna k, $\boldsymbol{x} = \left[x_1, \ldots, x_K\right]^{\mathsf{T}}$ is the column vector of complex channel input symbols x_k, where $\mathrm{E}\{\boldsymbol{x}\boldsymbol{x}^{\mathsf{H}}\} = \sigma_x^2 \boldsymbol{I}$ ($\mathrm{E}\{\cdot\}$: expectation, $(\cdot)^{\mathsf{H}}$: Hermitian transpose), likewise $\boldsymbol{n}$ the vector of additive white (complex) Gaussian noise samples n_k, mutually uncorrelated ($\mathrm{E}\{\boldsymbol{n}\boldsymbol{n}^{\mathsf{H}}\} = \sigma_n^2 \boldsymbol{I}$), and $\boldsymbol{y}$ is the vector of received signals.

The V-BLAST detection algorithm [1] is a popular method to equalize the interference and provide parallel AWGN subchannels. The particular virtue of this algorithm is that it optimizes the *permutation of the subchannels* for minimum symbol error rate, given that each subchannel uses the same transmit power.

In the present work we are interested in systems where channel state information *available to the transmitter* (TX-CSI) enables spatial loading [2], [3], i.e., an adaption of the transmit signal to the current subchannel characteristics. CSI at the transmitter is naturally available in time-division duplex systems (provided the “ping-pong” time is small relative to the fading coherence time), otherwise it can be provided via some backward channel. Note that most realistic transmission scenarios require TX-CSI to adapt the transmitter signaling to the (useable) rank of the current MIMO channel. We will show that the *permutation found by the V-BLAST algorithm is suboptimum* in this case and give an algorithm to obtain the optimum solution.

Decision-Feedback Equalization — We apply a QR-type decomposition [4] to $\boldsymbol{H}$ to obtain $\boldsymbol{H}\boldsymbol{P} = \boldsymbol{F}^{\mathsf{H}}\boldsymbol{S}$, where $\boldsymbol{P}$ is a permutation matrix permuting the columns of $\boldsymbol{H}$, $\boldsymbol{F}$ unitary, and $\boldsymbol{S} = \left[s_{kl}\right]$ lower triangular. If we define $\boldsymbol{G} = \boldsymbol{diag}(1/s_{11}, \ldots, 1/s_{KK})$ and $\boldsymbol{B} = \boldsymbol{G}\boldsymbol{S}$, this operation can be described by the block diagram in Fig. 1.

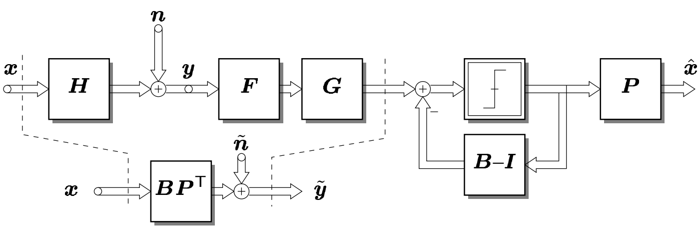

Fig. 1. Matrix DFE receiver with permuted subchannels.

Assuming error-free feedback, the variables, on which the decisions at the receiver are based, are only disturbed by white Gaussian noise $\tilde{\boldsymbol{n}} = \boldsymbol{G}\boldsymbol{F}\boldsymbol{n}$, $\mathrm{E}\{\tilde{\boldsymbol{n}}\tilde{\boldsymbol{n}}^{\mathsf{H}}\} = \sigma_n^2\,\boldsymbol{diag}(1/|s_{11}|^2, \ldots, 1/|s_{KK}|^2)$. In summary, the underlying MIMO channel is decomposed into K *parallel AWGN channels* with different noise variances $\sigma_n^2/|s_{kk}|^2$, $k = 1, \ldots, K$.

Calculating the Optimum Filter Matrices — If we perform *spatial loading*, i.e., distribute rate and power over the subchannels for maximum sum-capacity, the optimum power allocation is given by the *water filling rule*, which allocates transmit power

$$S_k = \sigma_x^2 + \frac{1}{K}\sum\nolimits_{\kappa=1}^{K} \frac{\sigma_n^2}{|s_{\kappa\kappa}|^2} - \frac{\sigma_n^2}{|s_{kk}|^2} \qquad (1)$$

to subchannel k (assuming all S_k's to be positive). The sum-capacity of this K-channel scheme is hence given as

$$\begin{aligned} C &= \sum_{k=1}^{K} \log_2\left(1 + \frac{S_k}{\sigma_n^2/|s_{kk}|^2}\right) \\ &= \log_2\left(\left(\frac{\sigma_x^2}{\sigma_n^2} + \frac{1}{K}\sum_{k=1}^{K}\frac{1}{|s_{kk}|^2}\right)^K \prod_{k=1}^{K}|s_{kk}|^2\right) . \end{aligned} \qquad (2)$$

Since $\boldsymbol{F}\boldsymbol{H} = \boldsymbol{S}\boldsymbol{P}^{\mathsf{T}}$, with $\boldsymbol{F}$ unitary, we have

$$\det(\boldsymbol{H}) = \det(\boldsymbol{F}\boldsymbol{H}) = \det(\boldsymbol{S}\boldsymbol{P}^{\mathsf{T}}) = \left(\prod\nolimits_{k=1}^{K} s_{kk}\right)\det(\boldsymbol{P}^{\mathsf{T}}) ,$$

and with $\det(\boldsymbol{P}^{\mathsf{T}}) = \pm 1$, the product $\prod_{k=1}^{K}|s_{kk}|^2$ is invariant with respect to the permutation matrix $\boldsymbol{P}$.

In order to maximize Eq. (2), we have to perform the factorization of $\boldsymbol{H}$ such that the sum of the inverse squares of the diagonal matrix entries s_{kk} is maximized (minimize the harmonic mean of the $|s_{kk}|^2$), i.e., the optimization problem is given as

$$\boldsymbol{P}_{\mathrm{opt}} = \underset{\boldsymbol{P}}{\operatorname{argmax}} \sum\nolimits_{k=1}^{K} \frac{1}{|s_{kk}|^2} . \qquad (3)$$

Looking at $\boldsymbol{B} = \boldsymbol{G}\boldsymbol{F}\boldsymbol{H}\boldsymbol{P} \stackrel{\mathrm{def}}{=} \boldsymbol{W}\boldsymbol{H}\boldsymbol{P}$, since $\boldsymbol{B}$ has a unit diagonal, i.e., $b_{11} = 1$ and additionally $b_{1l} = 0$ for $l > 1$, we realize that the first row of the matrix $\boldsymbol{W}$, denoted as $\boldsymbol{w}^1$, is just one row of the (pseudo) inverse of $\boldsymbol{H}$. Considering that $\boldsymbol{W} = \boldsymbol{G}\boldsymbol{F}$, with $\boldsymbol{F}$ unitary, the length of row k is equal to $1/|s_{kk}|$. Hence, the row of *maximum* length in the pseudo-inverse of $\boldsymbol{H}$ maximizes one term of the sum in (3). We select the matrix $\boldsymbol{P}$ s.th. the column of $\boldsymbol{H}$ corresponding to this row is permuted to the left. Deleting this column from the matrix $\boldsymbol{H}$, and forming the pseudo-inverse of this modified matrix, we get possible vectors for the second row $\boldsymbol{w}^2$. Continuing this way, constantly updating $\boldsymbol{P}$, the decomposition of $\boldsymbol{H}$ is constructed. Since the rows of the pseudo-inverse only null out the columns that have not yet been permuted to the left side of the original $\boldsymbol{H}$, the matrix $\boldsymbol{B}$ really has the triangular structure we require.

Fig. 2 gives the pseudo-code of a branch-and-bound algorithm that finds the $\boldsymbol{W}$ and $\boldsymbol{P}$ matrices solving (3). Of particular importance is the expression marked (*), where $(\cdot)^+$ denotes the Moore-Penrose pseudo-inverse [4] of a matrix, here consisting of the columns $\boldsymbol{h}_j$ of $\boldsymbol{H}$ that have not yet been selected. In particular, $\boldsymbol{G}^{(1)} = \boldsymbol{H}^+$. The rows of this matrix $\boldsymbol{G}^{(i)}$ are denoted as $\boldsymbol{g}^{j,(i)}$, where the row numbers j correspond to the original column numbers of $\boldsymbol{H}$ (e.g., if $K = 4$ and row 2 of $\boldsymbol{G}^{(1)}$ was selected in the first iteration, $k_1 = 2$, the 3 rows of $\boldsymbol{G}^{(2)}$ are numbered $1, 3, 4$). The ith unit vector is denoted by $\boldsymbol{e}_i$.

The first solution the algorithm comes up with is used to obtain a first lower bound on the expression to be maximized ($\sum_{k=1}^{K}||\boldsymbol{w}^k||^2$). In subsequent steps only such permutations are examined, where the upper bound obtained by looking at the sum of the lengths of the rows in $\boldsymbol{G}^{(i)}$ plus those already in $\boldsymbol{W}$ is greater than the lower bound (note that row lengths in $\boldsymbol{G}^{(i)}$ can only decrease when columns are deleted from $\boldsymbol{H}$).

It turns out that the greedy selection of the longest row of $\boldsymbol{G}^{(i)}$ in each step is (nearly) optimum in the vast majority of cases. Hence, if we select $k_i = \mathrm{argmax}_j\{||\boldsymbol{g}^{j,(i)}||^2\}$, resulting in the simpler recursion shown in Fig. 3, the loss with respect to the full search in Fig. 2 is

function $decompose(\boldsymbol{H})$
Initialization:
$\boldsymbol{W} = \left[(\boldsymbol{w}^1)^\mathsf{T}, \ldots, (\boldsymbol{w}^K)^\mathsf{T}\right]^\mathsf{T} = \boldsymbol{0}$
$\boldsymbol{P} = \left[\boldsymbol{p}_1, \ldots, \boldsymbol{p}_K\right] = \boldsymbol{0}$
$\mathcal{LB} = 0$
call $null(1)$
return $\boldsymbol{W}_\mathrm{opt}$, $\boldsymbol{P}_\mathrm{opt}$

subroutine $null(i)$
$\mathcal{K}_{i-1} = \{1, \ldots, K\} \setminus \{k_\ell, \ell < i\}$
$\boldsymbol{G}^{(i)} = \left[\boldsymbol{h}_j\right]^+_{j \in \mathcal{K}_{i-1}}$ (*)
$\mathcal{UB} = \sum_{\ell<i} ||\boldsymbol{w}^\ell||^2 + \sum_{j \in \mathcal{K}_{i-1}} ||\boldsymbol{g}^{j,(i)}||^2$
if $\mathcal{UB} < \mathcal{LB}$ return
for $k_i \in \mathcal{K}_{i-1}$, sorted for decreasing $||\boldsymbol{g}^{k_i,(i)}||^2$
$\boldsymbol{w}^i = \boldsymbol{g}^{k_i,(i)}$
$\boldsymbol{p}_i = \boldsymbol{e}_{k_i}$
if $i < K$
call $null(i+1)$
else
$\mathcal{LB} = \mathcal{UB}$
$\boldsymbol{W}_\mathrm{opt} = \boldsymbol{W}$
$\boldsymbol{P}_\mathrm{opt} = \boldsymbol{P}$

Fig. 2. Pseudocode for optimum matrix decomposition algorithm when loading is applied.

subroutine $null(i)$
$\mathcal{K}_{i-1} = \{1, \ldots, K\} \setminus \{k_\ell, \ell < i\}$
$\boldsymbol{G}^{(i)} = \left[\boldsymbol{h}_j\right]^+_{j \in \mathcal{K}_{i-1}}$
$k_i = \mathrm{argmax}_{j \in \mathcal{K}_{i-1}} \{||\boldsymbol{g}^{j,(i)}||^2\}$
$\boldsymbol{w}^i = \boldsymbol{g}^{k_i,(i)}$
$\boldsymbol{p}_i = \boldsymbol{e}_{k_i}$
if $i < K$
call $null(i+1)$
else
$\boldsymbol{W}_\mathrm{opt} = \boldsymbol{W}$
$\boldsymbol{P}_\mathrm{opt} = \boldsymbol{P}$

Fig. 3. Subroutine $null(i)$ for heuristic matrix decomposition algorithm for systems with loading.

negligible. Note that this algorithm resembles the V-BLAST algorithm if argmax is replaced by argmin.

Numerical Results — In Fig. 4 we show simulated error rates for a $K = 4$ Rayleigh fading channel, using M-QAM constellations with $M = 2^n$, $n = 2, \ldots, 8$, and the loading algorithm of [3] for spatial loading, to achieve a constant rate of 16 bit per channel use. We see that the optimized algorithms exibit practically identical gains in power efficiency of about $1.5\,\mathrm{dB}$ to more than $2\,\mathrm{dB}$ with respect to the V-BLAST algorithm applied to a system with loading, and immense gains compared to a system without loading.

Conclusion — Note that the results apply to other MIMO transmission scenarios, as well, provided CSI is available to the transmitter and loading is performed. The availability of CSI also enables the use of non-linear Tomlinson-Harashima precoding (THP) [5], and due to the duality of THP and DFE, the algorithms given here can also be used to calculate filter matrices for THP with loading across the dimensions.

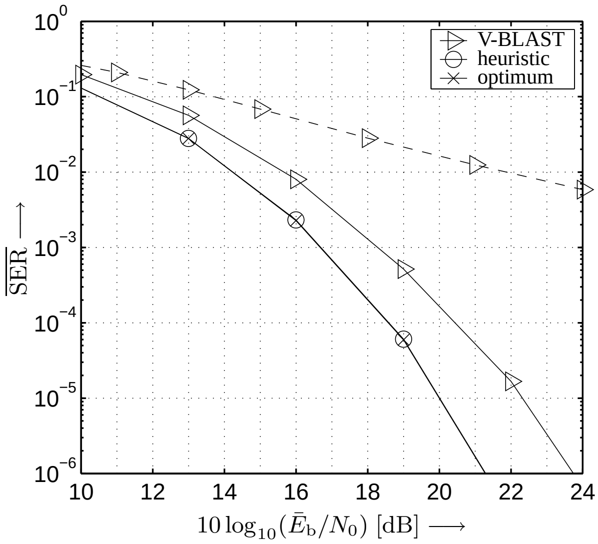

Fig. 4. Simulated average symbol error rates for DFE with (–) and without (- -) loading.

REFERENCES

[1] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolniansky. Simplified Processing for High Spectral Efficiency Wireless Communication Employing Multi-Element Arrays, *IEEE Journal on Selected Areas in Communications*, JSAC–17, pp. 1841–1852, November 1999.

[2] A. J. Goldsmith and P. P. Varaiya. Capacity of Fading Channels with Channel Side Information *IEEE Trans. Commun.*, vol. 43, pp. 1986–1992, November 1997.

[3] R. F. H. Fischer and J. B. Huber. A New Loading Algorithm for Discrete Multitone Transmission, In *IEEE Global Telecommunications Conference*, pp. 724-728, London, November 1996.

[4] G. H. Golub and C. F. Van Loan. *Matrix Computations*, 3rd Edition, The Johns Hopkins University Press, Baltimore, 1996.

[5] R. F. H. Fischer, C. Windpassinger, A. Lampe, and J. B. Huber. Space-Time Transmission using Tomlinson-Harashima Precoding, *4th International ITG Conference on Source and Channel Coding*, Berlin, January 2002.