Noncoherent Adaptive Linear MMSE Equalization for
16DAPSK Signals

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Abstract

In this letter, a novel adaptive noncoherent linear minimum mean-squared error (MMSE) equalizer for 16-level differential amplitude/phase-shift keying (16DAPSK) signals is proposed. A novel modified least-mean-square (LMS) algorithm is derived which allows an efficient equalizer adaptation. Simulations confirm the high performance of the proposed noncoherent equalizer and its robustness against carrier phase variations.

1 Introduction

Recently, 16-level differential amplitude/phase-shift keying (16DAPSK) has received high attention (see e.g. [1, 2, 3, 4] and references therein). Both, the amplitude and the phase of 16DAPSK symbols are differentially encoded and thus, a robust noncoherent amplitude and phase detection is possible. The disadvantage of conventional (noncoherent) differential detection (DD) [1] is a high loss in power efficiency compared to coherent detection (CD). Therefore, for frequency–nonselective channels several improved noncoherent detectors have been proposed (e.g. [2, 3, 4]). For transmission over frequency–selective channels, however, no noncoherent receiver has been reported so far. Note, that conventional (coherent) equalizers [5] are very sensitive to frequency offset as will be shown in our computer simulations. Thus, we propose a novel noncoherent MMSE equalizer which takes full advantage of the differential encoding of 16DAPSK and is robust against phase variations.
2 Transmission Model

In this letter, all signals are represented by their complex-valued baseband equivalents. For simplicity, only $T$-spaced equalizers are considered here. However, our results can be extended easily to the fractionally-spaced case. The transmitted 16APSK symbol $s[k]$ is given by

$$s[k] \triangleq R[k] b[k], \quad k \in \mathbb{Z},$$

with absolute amplitude symbol $R[k]$, $R[k] \in \{ R_L, R_H \}$, $R_H > R_L$, and absolute phase symbol $b[k] \in \mathcal{A}_b = \{ e^{j2\pi/4} \nu | \nu \in \{0, 1, \ldots, 7\} \}$. For convenience, $\mathcal{E}\{ |s[k]|^2 \}$ ($\mathcal{E}\{ \cdot \}$ denotes expectation) is normalized to unity. $s[k] = \Delta s[k] s[k-1]$ is obtained from the 16APSK symbol $\Delta s[k] \triangleq \Delta R[k] a[k]$ via differential encoding [1]. $\Delta R[k] = R[k]/R[k-1] \in \mathcal{A}_{\Delta R} = \{ a_0, 1, 1/a_0 \}$, where $a_0 = R_H/R_L$ denotes the ring ratio, and $a[k] = b[k]/b[k-1] \in \mathcal{A}_b$ refer to the amplitude and the phase difference symbol, respectively. One information bit is mapped to the amplitude difference symbol, while three information bits are Gray-mapped to the phase difference symbol. $\Delta R[k] = 1$ and $\Delta R[k] = a_0$ (if $R[k-1] = R_L$) or $\Delta R[k] = 1/a_0$ (if $R[k-1] = R_H$) is valid if the information bit is equal to zero and one, respectively.

The 16APSK symbols $s[k]$ are transmitted over an intersymbol interference (ISI) channel with unknown, constant phase shift $\Theta$ and normalized frequency offset $\Delta f T$ ($T$ denotes the symbol duration). The discrete-time received signal, sampled at times $k T$ at the output of the receiver input filter, can be modelled as

$$r[k] = e^{j\Theta} e^{j2\pi/TT_k} \sum_{\nu=0}^{L_k-1} h_\nu s[k-\nu] + n[k],$$

where $h_\nu$, $0 \leq \nu \leq L_k - 1$, are the coefficients of the combined discrete-time impulse response of the cascade of transmit filter, channel, and receiver input filter; its length is denoted by $L_k$.

For the receiver input filter, we assume a square-root Nyquist frequency response. Thus, the zero mean complex Gaussian noise $n[\cdot]$ is white. Due to an appropriate normalization, the noise variance is $\sigma_n^2 = \mathcal{E}\{ |n[k]|^2 \} = N_0/E_S$. $E_S$ and $N_0$ are the mean received energy per symbol and the single-sided power spectral density of the underlying passband noise process, respectively.

3 Receiver Structure

Fig. 1 shows the proposed receiver structure. Similar to noncoherent linear equalization schemes reported for MDPSK [6, 7], we propose to place the equalizer before the noncoherent (nonlinear) detector. This has the great advantage that only linear ISI has to be equalized. The equalizer
output symbol $q[k]$ may be written as

$$q[k] = \sum_{\nu=0}^{L_e-1} c_\nu[k] r[k-\nu],$$

(3)

where $c_\nu[k]$ are the equalizer coefficients; $L_e$ denotes the equalizer length.

The next stage of the proposed receiver is a decision-feedback differential detector (DF-DD) [2], which determines an estimate $\Delta \hat{s}[k-k_0] = \Delta \hat{R}[k-k_0] \hat{a}[k-k_0]$ for the transmitted symbol $\Delta s[k-k_0]$ based on a reference symbol $q_{\text{ref}}[k-1]$. Here, $k_0$ is an appropriate decision delay. The reference symbol may be generated either recursively [2]

$$q_{\text{ref}}[k-1] = \frac{1}{N-1} \sum_{\nu=0}^{N-1} q[k-\nu] \prod_{\mu=1}^{\nu-1} |\Delta s[k-k_0-\mu]|$$

(4)

($(^\ast)$ denotes complex conjugation), where $N$, $N \geq 2$, is the number of equalizer output symbols used for determination of $\Delta \hat{s}[k-k_0]$, or recursively [2]. Because of space limitations, here, we restrict ourselves to the nonrecursive case. Using the decision variable $d[k] = q[k]/q_{\text{ref}}[k-1]$, the estimated amplitude difference symbol $\Delta \hat{R}[k-k_0]$ and phase difference symbol $\hat{a}[k-k_0]$ may be determined as proposed in [2].

4 Equalizer Adaptation

For equalizer adaptation a modified LMS algorithm is derived. For this a proper (noncoherent) error criterion, which guarantees insensitivity against phase variations, has to be found. One possible error criterion for a modified LMS algorithm would be $e'[k] = \Delta s[k-k_0] - d[k]$. However, simulations showed that the corresponding adaptive algorithm is unstable even for very small adaptation step sizes. Thus, here, we use the error signal $e[k] = a[k-k_0] - q[k]q_{\text{ref}}[k-1]$. The modified LMS algorithm is obtained from [5, 8]

$$c[k+1] = c[k] - \delta_{\text{LMS}} \frac{\partial}{\partial e[k]} |e[k]|^2$$

(5)

where the definition $c[k] \overset{\triangle}{=} [c_0[k] \ c_1[k] \ldots \ c_{L_e-1}[k]]^H$ ($[^H]$ denotes Hermitian transposition) is employed and $\delta_{\text{LMS}}$ is the adaptation step size. Using Eq. (3), the definition $r[k] \overset{\triangle}{=} [r[k] \ r[k-1] \ldots \ r[k-L_e+1]]^T$ ($[^T]$ denotes transposition), and taking into account that $q_{\text{ref}}[k-1]$ depends only on $c[k-\nu], \nu \geq 1$, which have to be treated as constants for differentiation with respect to $c[k]$ (cf. [6, 7]), Eq. (5) may be rewritten to

$$c[k+1] = c[k] + \delta_{\text{LMS}} e^* [k] r[k] q_{\text{ref}}^*[k-1].$$

(6)
where the method for complex differentiation described in Appendix B of [8] is used. An analytical performance analysis of the proposed modified (noncoherent) LMS algorithm is very difficult if not impossible and beyond the scope of this letter. Our simulations showed that it has a similar convergence speed like the conventional (coherent) LMS algorithm. Moreover, the novel algorithm always converged to the desired solution. The step size parameter has to fulfill similar conditions for stability like in the coherent case.

5 Simulation Results

Fig. 2 shows the bit error rate (BER) vs. $10 \log_{10}(E_b/N_0)$ ($E_b = E_z/4$ is the mean received energy per bit) for the proposed noncoherent receiver with nonrecursively generated reference symbol (cf. Eq. (4)). Here, the ring ratio $a_0 = 2.0$ and channel impulse response coefficients $h_0 = 0.304$, $h_1 = 0.903$, $h_2 = 0.304$ (specified in [5]) are used. For comparison, Fig. 2 also contains the BER caused by a conventional (coherent) MMSE equalizer [5] employing a conventional LMS algorithm and CD. In all cases, an equalizer length of $L_c = 7$, a decision delay of $k_0 = 4$, and a step size of $\delta_{\text{LMS}} = 0.01$ are employed. A training sequence is transmitted until the equalizer operates in steady state; then a transition to the decision-directed mode occurs. It can be observed, that for zero frequency offset ($\Delta fT = 0$, solid lines) the proposed noncoherent equalizer approaches the coherent equalizer for $N \gg 1$. On the other hand, for $\Delta fT = 0.005$ (dashed lines) the coherent receiver degrades severely since the conventional LMS algorithm cannot track the phase variations caused by the frequency offset. Note, that this is also true if $\delta_{\text{LMS}}$ is increased. The novel noncoherent equalizer degrades the less the smaller $N$ is chosen. Clearly, there is a trade-off between robustness against frequency offset and power efficiency for $\Delta fT = 0$ which is typical for noncoherent receivers (cf. e.g. [3]). Thus, in practice, $N$ has to be optimized with respect to the maximum expected frequency offset.

6 Conclusions

A novel noncoherent linear MMSE equalizer for 16DAPSK signals transmitted over ISI channels has been proposed and an efficient modified LMS algorithm for equalizer adaptation has been derived. The simulations presented confirm that the proposed noncoherent equalizer is far more robust against frequency offset than conventional coherent equalizers. In a practical system the design parameter $N$ has to be chosen carefully since it enables a trade-off between power efficiency
for zero frequency offset and robustness against phase variations. Although, in this letter, we restricted ourselves to 16DAPSK, the proposed scheme may also be applied to other DAPSK modulation formats. It should be mentioned that it is also possible to derive a fast converging modified recursive least-squares (RLS) algorithm for equalizer adaptation. However, because of space limitations, here, only the modified LMS algorithm was described.

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References


Figure Captions

Figure 1: Proposed receiver structure for the decision-directed mode.
Figure 2: BER vs. $10 \log_{10}(E_b/N_0)$ for the proposed noncoherent equalizer.

- - - $\Delta fT = 0$
- - - $\Delta fT = 0.005$

$\nabla$: $N = 2$, $\Box$: $N = 3$, $\triangle$: $N = 5$, $\times$: $N = 10$, $*$: coherent MMSE equalizer.