Using Flexible Precoding for Channels with Spectral Nulls

Robert Fischer
Lehrstuhl für Nachrichtentechnik, Universität Erlangen–Nürnberg
Cauerstrasse 7, D-91058 Erlangen, Germany
Tel.: +49 9131 857114 FAX: +49 9131 303840
e-mail: fischer@nt.e-technik.uni-erlangen.de

Abstract — Flexible precoding was proposed for channel pre-equalization at the transmitter side. In this paper, a slight modification of the receiver is presented which enables the use of flexible precoding for channels with spectral nulls at DC and/or the Nyquist frequency. Simulation results covering the performance are given.

1 Introduction

For fast digital transmission, precoding at the transmitter has become a popular technique to eliminate intersymbol interference (ISI) caused by bandlimited channels. Using precoding, coded modulation can be applied in a straightforward manner. Since linear pre-equalization boosts transmit power, Tomlinson-Harashima precoding (TH precoding) [1] applies a nonlinear reduction of transmit signal amplitude. Unfortunately, TH precoding cannot be combined with any shaping technique because it is not transparent to signal shaping.

In [2, 3] a new precoding scheme, called Flexible Precoding is proposed. It is transparent to shaping and can easily be combined with channel coding, developed for the channel without ISI. Because the inverse channel filter has to be realized at the receiver, flexible precoding cannot be applied to channels with spectral nulls. In this paper, we give a slight modification of flexible precoding for channels with zeros at DC and/or the Nyquist frequency.

2 Flexible Precoding – State of the Art

Fig. 1 shows the block diagram of the transmission model applied in this paper. The discrete-time channel model, denoted by \( H(z) = \mathcal{Z}\{h(k)\} \), is assumed to be the transfer function of a monic, minimum-phase FIR filter of order \( p \).

In the precoder, a sequence \( \langle x(k) \rangle (k \in \mathbb{Z}) \) of channel symbols is generated from the sequence \( \langle a(k) \rangle \) of data symbols. If coded modulation is applied, \( \langle a(k) \rangle \) is a valid

\(^1\mathcal{Z}\{\cdot\}: z\text{-transform}\)
sequence from the underlying trellis code, which is assumed to be based on a partitioning of the signal constellation into cosets, consisting of translates of $\Lambda_c$, the so called “coset lattice”.

Let $x(k) = a(k)$ be transmitted over the channel without precoding. Then, the channel output $v(k)$ is $a(k)$ plus $f(k) = \sum_{i=1}^{P} h(i) \cdot x(k-i)$, the postcursors of prior transmitted symbols. Using $H(z) - 1$ the sequence $\langle f(k) \rangle$ can also be generated at the transmitter. Recall, that the sum of a trellis coded sequence and a sequence, with elements taken exclusively from the coset lattice $\Lambda_c$, is also a trellis code sequence. Hence, $v(k)$ would be a valid signal point if $f(k)$ lies in the coset lattice $\Lambda_c$. Since this is not true in general, $f(k)$ is quantized to the nearest point $d(k) \in \Lambda_c$ and the quantization error $m(k) = f(k) - d(k)$ is subtracted from the data symbol. This ensures $v(k) = x(k) + f(k) = a(k) - m(k) + d(k) + m(k) = a(k) + d(k)$ to be an element of a trellis coded sequence. In Fig. 1 the determination of $m(k)$ is shown as a modulo $\Lambda_c$ operation. Because of the independence of $a(k)$ and $m(k)$, $x(k)$ lies in the region $\mathcal{R}$ given by the support of the convolution of the corresponding densities.

At the receiver an estimate $\hat{v}(k)$ is generated by a trellis decoder. In order to recover data, $\hat{v}(k)$ is filtered with the inverse channel filter $1/H(z)$ to get an estimate $\hat{x}(k)$ of the transmit symbols. Because $m(k)$ is in the Voronoi region of $\Lambda_c$ and $a(k)$ and $v(k)$ lie in the same coset, data can be recovered by quantizing $\hat{x}(k)$ to the nearest point in the current coset. This operation is performed at the block labeled “inverse precoder”.

Since the inverse of the channel filter is needed at the receiver, it becomes clear that $H(z)$ has to be strict minimum-phase. Zeros at the unit circle, i.e. spectral nulls, lead to spectral poles in $1/H(z)$ and the inverse channel filter no longer is stable.

### 3 Modifying the Receiver

Our aim is to modify solely the receiver of the flexible precoding scheme in order to use this technique for the class of channels with zeros at $z = 1$ and/or $z = -1$. In order to find such a modification we at first study the effects caused by an unstable filter. If there is no channel noise or the trellis decoder works perfect the spectral poles have no effect at all, because the sequence $\langle x(k) \rangle$ to be recovered is initially filtered by $H(z)$. Hence, the zeros of $H(z)$ and the poles of $1/H(z)$ cancel each other and the transmission is stable. Consequently, only the effect of decision errors has to be studied, which can be done separately, as we deal with linear filters.

Decision errors are characterized by the corresponding error sequence $\langle e_v(k) \rangle$ with $e_v(k) = \hat{v}(k) - v(k)$, which is assumed to be limited in time. The elements $e_v(k)$ are differences between valid signal points. First, let us suppose $H(z)$ has a zero at $z = 1$. Filtering $\langle e_v(k) \rangle$ by $1/H(z)$, due to the integrating part, in the steady state a constant sequence $\langle e_x(k) \rangle$ with $e_x(k) = \hat{x}(k) - x(k)$ of infinite duration results. If $H(z)$ has a zero at $z = -1$, $\langle e_x(k) \rangle$ is an alternating sequence with constant envelope.
Now, the question is how to detect $\langle e_x(k) \rangle$. Recall, that the transmit symbols $x(k)$ are restricted to the region $\mathcal{R}$. If the recovered symbol $\hat{x}(k)$ exceeds this area a decision error must have occurred. Then, the error sequence $\langle e_x(k) \rangle$ can be compensated by feeding an additional impulse into the filter $1/H(z)$. Because in the steady state the response must be the negative of $\langle e_x(k) \rangle$, the correction impulse also has to have the properties of an error sequence. The current amplitude is determined by the amount and direction $\hat{x}(k)$ exceeds $\mathcal{R}$ in order the corrected $\hat{x}(k)$ lies inside $\mathcal{R}$.

Similar to TH precoding, the stabilization of $1/H(z)$ can be described using a nonlinear device, shown in Fig. 2. Here, for conciseness the one-dimensional $M$-ary signal set $a(k) \in \{\pm 1, \pm 3, \ldots, \pm (M - 1)\}$ and uncoded transmission are assumed, hence $\mathcal{R} = [-M, M)$. For small signals, the nonlinearity has linear course. Signals with amplitude outside $\mathcal{R}$ are reduced to this range using a saw-tooth characteristic with step size equal to the signal point spacing. The nonlinear device ensures $\hat{x}(k)$ to be bounded — the inverse channel filter is stable. If coded modulation and flexible precoding as described in [2] are applied $\mathcal{R}$ has to be fitted to the current coset.

Recently, modifications of flexible precoding resulting in a smaller precoding loss were proposed [3]. Since they need $1/H(z)$ at the receiver, too, the presented methods of using flexible precoding for channels with spectral nulls at $z = \pm 1$ can be used as well. If QAM transmission is desired, i.e. $H(z)$ has complex coefficients, the nonlinearity is simply applied to the real and imaginary part of $\hat{x}(k)$.

### 4 Examples and Conclusions

The proposed modification is shown exemplarily for a fourth-order channel with zeros at $z = \pm 1$ and $z = 0.9 \cdot \exp\{\pm \frac{2}{3} \pi\}$. Fig. 3 summarizes the symbol error rate versus the signal-to-noise ratio $E_b/N_0$. The solid lines correspond to flexible precoding using the nonlinear receiver filter. For reference, TH precoding is used for transmission as well (dashed lines). In each case, uncoded 4-ary signaling and coded 8-ary transmission using the 4-state Ungerboeck code [4] are shown. It is noticeable that due to error propagation in $1/H(z)$ the error rate of flexible precoding is about ten times that of TH precoding. (This effect is recognizable for channels without zeros on the unit circle, too.) But, in spite of the spectral nulls the receiver remains stable.

Applying the modification given in this paper, flexible precoding can be extended to the wide class of channels with zeros at DC and/or the Nyquist frequency. The additional complexity is negligible and the advantages of flexible precoding are not affected. Consequently, the restriction to strict minimum-phase channels claimed in [2, 3] can be dropped.
References


Figure 1: Block diagram of the transmission model.

Figure 2: Stable version of the inverse channel filter.

Figure 3: Symbol error rate versus the signal-to-noise ratio $E_b/N_0$. 

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