Comparison of Code Design Requirements for Single- and Multicarrier Transmission over Frequency-Selective MIMO Channels

Robert F.H. Fischer, Clemens Stierstorfer
Lehrstuhl für Informationsübertragung, Friedrich–Alexander–Universität Erlangen–Nürnberg,
Cauerstrasse 7/LIT, 91058 Erlangen, Germany, Email: {fischer,clemenss}@LNT.de

Abstract—Point-to-point transmission over multiple-input/multiple-output channels with intersymbol interference is considered. Two different equalization strategies—singlecarrier transmission with spatial/temporal decision-feedback equalization and multicarrier transmission with V-BLAST in each carrier—and the respective requirements on channel coding are studied and compared. Via capacity arguments, it is shown that bit-interleaved coded modulation is not well suited for the situations hand. This holds in particular if channel coding is used in combination with rate loading, i.e., non-uniform rate distribution at hand. This works was supported by Deutsche Forschungsgemeinschaft (DFG) under grant HU 634/5–1

I. INTRODUCTION

Meanwhile it is well-known that using multiple-input/multiple-output (MIMO) transmission, spectral efficiency and thus data rate can be dramatically increased. Mostly, flat-fading MIMO channels are assumed. However, using larger bandwidths for further increasing data rates, the frequency selectivity of the channels becomes more and more important. Then, in addition to multiuser interference (MUI, spatial interference) of the parallel data streams, intersymbol interference (ISI, temporal interference) is additionally present (MIMO ISI channel).

This paper compares the requirements on channel coding for point-to-point transmission over MIMO ISI channels. Thereby, two different equalization strategies are assumed: Multicarrier modulation (orthogonal frequency division multiplexing, OFDM) in combination with MIMO transmission techniques in each carrier, and singlecarrier modulation with joint spatial/temporal decision-feedback equalization (DFE).

In Section II, the channel model is given. Section III briefly explains MIMO OFDM and MIMO DFE, respectively, and reviews their equivalence in case of rate and power loading. In Section IV, the requirements on the code design in both scenarios are discussed. Thereby the role of channel state information and of loading is addressed. Via the capacities of the individual coding levels it is shown why bit-interleaved coded modulation (BICM) does not show the usually expected good performance in the present situations.

II. CHANNEL MODEL

In this paper we assume point-to-point transmission over a MIMO channel with ISI using K transmit and R receive antennas (the results are easily generalized). Over each antenna a sequence of T-spaced data symbols taken from a signal constellation (variance $\sigma_n^2$) is transmitted.

We assume a T-spaced discrete-time channel model (given in the equivalent complex baseband domain [13] and comprising pulse shaping at the transmitter, the actual continuous-time channel, matrix matched filtering, T-spaced sampling, and suited discrete-time processing at the receiver), whose transfer function is given by the matrix polynomial (L: length of the impulse response) $H(z) = \sum_{k=0}^{L-1} H_k z^{-k}$, with $K \times K$ tap matrices $H_k$. The channel is considered to be block fading, i.e., during each transmission burst the channel transfer function is constant. The additive discrete-time Gaussian noise is assumed to be spatially and temporally white with variance $\sigma_n^2 = \frac{\sigma_0^2}{2}$ per complex component.

III. SINGLE- AND MULTICARRIER TRANSMISSION

Basically there are two concepts for point-to-point transmission over MIMO ISI channels, which can be distinguished in the way how they deal with MUI (spatial equalization) and ISI (temporal equalization).

A. Multicarrier Transmission

First, MUI and ISI can be treated separately, i.e., different and independent equalization strategies are applied to both types of interferences. Multicarrier modulation is very popular in transmission over dispersive channels [2]. Here, temporal equalization is performed by appropriate scaling of the tones in frequency domain.

In MIMO systems temporal equalization can be performed by applying an OFDM transmitter (signal mapping in frequency domain; inverse discrete Fourier transform (DFT); guard interval insertion) and OFDM receiver (guard interval removal; discrete Fourier transform; detection/decoding in frequency domain) to each transmit and receive antenna, respectively (MIMO OFDM). Assuming a guard interval of sufficient length, the MIMO ISI channel is decomposed into a set of parallel, independent flat fading MIMO channels. Denoting the length of the DFT by $D$, the channel matrix of the MIMO channel at carrier $d$, $d = 1, \ldots, D$, is related to the transfer function of the underlying channel by

$$H^{(d)} = H (e^{j2\pi(d-1)/D}), \quad d = 1, \ldots, D. \quad (1)$$

The additive noise in each carrier remains white with correlation matrix $\sigma_n^2 I$ ($I$: Identity matrix).
Then, for each of this parallel channels (the carriers), spatial equalization is performed individually. This can be done by using any of the well-known detection techniques for flat fading MIMO channels. Of particular interest are singular value decomposition (SVD) and the V-BLAST detection scheme, which implements the decision-feedback principle.

### B. Singlecarrier Transmission

Second, MUI and ISI can be treated jointly by performing combined spatial/temporal decision-feedback equalization (MIMO DFE).

For spatial/temporal DFE, the received signal is passed through a matrix feedforward filter \(F(z)\), which (i) shapes the end-to-end signal transfer function \(B(z) = \sum_k B_k z^{-k} \) to exhibit spatial and temporal causality and (ii) produces spatially and temporally uncorrelated noise samples. Symbols transmitted at time instant \(k\) should only interfere into receive symbols at time instances \(\kappa > k\) and, in addition, symbols transmitted at the same time instance over antenna \(\mu\) should only interfere into receive symbols for antennas \(m > \mu\) (assuming an optimized permutation). Then, the symbols over space and time can be processed in a zigzag fashion. Moreover, since the decisions are based on the diagonal elements of the zeroth end-to-end coefficient—all other taps are canceled,—these elements should be as large as possible.

Mathematically, \(B(z)\) has to be causal, the zeroth coefficient, \(B_0\), (assuming proper scaling) has to be lower triangular with unit main diagonal, and it has to be minimum phase (\(\det(B(z)) \neq 0, |z| > 1\), and an optimized detection order is chosen. The required matrix filters are obtained by performing a spectral factorization, see [16], [9].

### C. Comparison of Uncoded Transmission

Applying these equalization strategies, the MIMO ISI channel is decoupled into a set of parallel, independent (sub-)channels. In MIMO DFE, \(K\) parallel subchannels, used at every time instant \(k\), are present, whereas in OFDM (using all carriers) \(D \cdot K\) subchannels, usable once in each block of \(D\) consecutive symbols, are available. In each case, the subchannels have individual SNRs, and hence for uncoded transmission using the same constellations and same powers the worst case subchannel will dominate error rate.

In such situations, system performance can be improved (significantly) by applying an optimized, nonuniform distribution of total rate and total power to the parallel subchannels, called loading. In the context of OFDM a number of practical loading algorithms have been proposed, e.g., [6]. Using optimal loading (ignoring the unavoidable quantization of the rate), the geometric mean of the individual SNRs takes effect [6].

It is easy to show that in case of optimal loading MIMO DFE and MIMO OFDM achieve the same SNR [8] \((H_0(z) = H^H(z^{-1})H(z))\)

\[
\text{SNR} = \frac{\sigma_n^2}{\sigma_a^2} \exp \left\{ \frac{1}{K} \int_0^1 \log|\det(H_0(e^{j2\pi\phi}))| \ d\phi \right\}, \tag{2}
\]

and hence exhibit the same error rate. This equivalence is based on the assumption that the loss due the guard interval is ignored in OFDM. Moreover, the effect of error propagation in the V-BLAST and MIMO DFE detection loops is neglected as well.

In summary, the theoretical equivalence reveals that both strategies have the same potential. In uncoded transmission, rate and power loading is the key to achieve this performance.

### IV. COMPARISON OF CODE DESIGN

In practical schemes, the above equalization approaches will be combined with channel coding. In this section we discuss and compare the different requirements on the channel coding schemes usable for MIMO DFE or MIMO OFDM, respectively. We assume that coding is restricted to a transmission burst; no coding over different channel realizations is performed. During each burst, the channel is time-invariant and given.

#### A. Channel State Information

For optimum detection/decoding of the transmitted symbols, channel state information (CSI) is required at the receiver. Throughout this paper, we assume perfect CSI at the receiver (RX CSI). For uncoded transmission, using MIMO DFE or MIMO OFDM with V-BLAST in each carrier, RX CSI is sufficient. However, when using loading, some degree of CSI is also required at the transmitter side (TX CSI). The receiver may calculate rate and power distribution which then has to be communicated to the transmitter.

1) Multicarrier Transmission: From the theory of (perfectly interleaved) flat-fading channels it is known that capacity approaching channel coding can be done by applying a single channel code with appropriate rate [1]. The code itself averages over the different fading states. For ideal codes, CSI at the transmitter would offer no or only little advantage. The same is true when coding across the carriers in a multicarrier scheme [3].

Hence, in contrast to uncoded transmission where TX CSI is essential [8], similar performance of coded transmission over MIMO ISI channels may be expected for single- and multicarrier approaches even without TX CSI available. However, for MIMO ISI channel there are points beyond to be considered: First, if no TX CSI is available, MIMO OFDM is only possible in combination with V-BLAST in each carrier. The application of SVD already requires TX CSI. Channel coding can be combined with V-BLAST by coding over each of the \(K\) antennas (parallel data streams) separately, visualized in Fig. 1. Having received an OFDM symbol, after transform to frequency domain and feedforward filtering per carrier, the codewords are successively decoded and then their interference to other layers is subtracted. This procedure is sometimes called horizontal (H) BLAST.

Note, if no TX CSI is available, no optimized order of the data streams per carrier is possible. Hence, over each antenna, an individual codeword is transmitted. At the receiver only one global (the same for all carriers; per codeword) order of decoding is possible, i.e., an optimized order per carrier...
is prevented (see, e.g., [11]). If sorting is done per carrier, the codewords transmitted in parallel experience different channel conditions. The word decoded first (on the average) is transmitted over a channel with lowest SNR, whereas the word decoded last—due to cancellation of the interference of the other words (assuming correct decisions)—has best conditions. Thus, the codewords cannot average over the different channel conditions, and, ideally, codes with different, adopted rates should be used, which, in turn, requires TX CSI.

If TX CSI is available, MIMO OFDM can be combined with SVD per carrier. The main advantage is that in this case no decisions for the cancellations of interference are required. Therefore, a single codeword of length $DK$ can be arranged over space (antennas) and frequency.

2) Singlecarrier Transmission: The situation changes when singlecarrier transmission is considered. First, channel coding cannot be combined straightforwardly with MIMO DFE since immediate decisions are required. As in SISO transmission this may only be overcome by clever interleaving techniques [5]. The solution is to replace DFE at the receiver by precoding at the transmitter, which, however, requires TX CSI. Precoding is basically obtained from DFE by flipping the entire transmission scheme; except minor differences, precoding basically performs the same as DFE and the required filters are obtained similar as given above. For details see, e.g., [7], [9].

Using precoding, a single codeword can be arranged over antennas and time, also visualized in Fig. 1. Since TX CSI is required anyway, an optimized processing order can be used. Now, $K$ parallel subchannels with individual SNRs are present—a periodically time-varying channel is active. In contrast to MIMO OFDM, in MIMO DFE/precoding the code has to average over only $K$ different channel conditions instead of $DK$ one. Moreover, due to the feedforward processing which does some sort of combining, the $K$ parallel subchannels have very similar SNRs. Hence the channel code does not need to average over different channel conditions.

3) Numerical Results: The different channel conditions present in MIMO OFDM and MIMO DFE/precoding, respectively, are now compared by means of numerical simulations. For that, we assume a $4 \times 4$ ($K = 4$) block-fading MIMO ISI channel with constant power-delay profile (equal-gain test channel, $L = 4$ $T$-spaced taps). The elements of the tap matrices $H_k$ are chosen i.i.d. complex Gaussian with variance $1/L$. The number of carriers in OFDM is chosen to $D = 256$; all carriers are used. The histogram of the noise powers (in dB) of the parallel subchannels are depicted in Fig. 2 for 16 channel realizations. The much larger spread of the $256 \cdot 4 = 1024$ subchannels of MIMO OFDM in comparison with the 4 subchannels of MIMO DFE/precoding is clearly visible.

![Fig. 2. Histogram of the noise powers (in dB) of the parallel subchannels in MIMO OFDM with V-BLAST (left) and MIMO DFE/precoding (right). Rows: 16 channel realizations.]

B. Combined Loading and Channel Coding

Rate and power loading can be combined with channel coding, which is expected to offer a performance gain. Since in practice no ideal channel coding is present, averaging over the channel states may be incomplete. Averaging the parallel subchannels via loading alleviates the requirements on the coding scheme which then basically operates over a non-fading AWGN channel. Now, TX CSI is indispensable.

A very attractive coded modulation scheme, often used in combination with OFDM is bit-interleaved coded modulation (BICM) [4]. We study BICM for use in loaded MIMO OFDM and MIMO DFE/precoding, respectively. For this we assume that Gray labeling of the signal constellation is done per quadrature component; individual Gray labeling in inphase and quadrature component immediately leads to Gray labeling of QAM signal sets. Moreover, one-dimensional Gray labeling for $M$-ary constellations is done as shown in [13, Page 171]. This type of Gray labeling preserves its property if the signal constellation is extended periodically, which is the case in precoding.

The design of the loading algorithm should be based on the properties of BICM rather than doing loading for uncoded transmission and putting channel coding on top. For this, the capacities of the individual binary coding levels (see, e.g., [15]) are numerically calculated. Since BICM and no
multistage decoding is employed, the capacities of “parallel decoding of levels” are of interest.

In Fig. 3 the capacity curves are shown for $M = 2, 4, 8, 16, 32$, and 64-ary one-dimensional constellations. In addition, the capacity limits using BICM with a rate-1/2 and a rate-2/3 code, respectively, are marked (dashed vertical lines). From this figure two main conclusions can be drawn: First, a loading algorithm has to take the sum capacity limits (usually with some gap) into account. The respective capacity limits are spaced differently compared to loading for uncoded one-dimensional constellations. Resorting, e.g., to the loading algorithm proposed in [12], the respective SNR values may easily be considered.

Second, going to higher-level modulation the variation of the capacities of the parallel coding levels increases significantly. At the sum capacity limit, some binary coding levels offer a capacity of more than 0.7 . . . 0.8 bits, whereas others show only a capacity of 0.1 bits or less. Using BICM in combination with “small constellations” works very well; the level capacities are very similar. However, using BICM for “large constellations”, some bits will be very unreliable.

Applying BICM without rate and power loading, the code has to average over the different subchannels produced by MIMO OFDM or MIMO DFE/precoding. However, using, e.g., 256-QAM signaling with rate-1/2 code for achieving a spectral efficiency of 2 bits/one-dimensional symbol (reference: uncoded 16-QAM), the code has to average over the coding levels which exhibit a large spread (using 64-QAM signaling with rate-2/3 code, which achieves the same spectral efficiency, leads to a much smaller spread). Conversely, using rate and power loading, the parallel subchannels produced by MIMO OFDM or MIMO DFE/precoding are balanced. However, since smaller and larger signal constellations are used at the same time, the code has to average over coding levels which exhibit a even larger spread. In both cases poor performance results.

In case of MIMO precoding the signal constellations are extended periodically. This in turn has an effect on the capacities of the binary coding levels. The capacity curves for $M = 2, 4, 8, 16, 32, 64$-ary one-dimensional constellations which are extended periodically are shown in Fig. 3, too. The capacities are somewhat decreased but the conclusions are the same as drawn above.

In summary it can be stated that the straightforward combination of MIMO OFDM or MIMO DFE/precoding with BICM will not give the desired performance. There are two possible strategies to alleviate this problem: First, a suited design of the bit interleaver present in BICM is required. Random bit interleaving in combination with convolutional codes will likely combine unreliable bits into one trellis segment or even consecutive segments. This will produce high error rates. Interleaving only within the binary coding levels should be preferred in MIMO OFDM, where averaging is required over the states of the underlying parallel subchannels. In MIMO DFE/precoding, where such an averaging is not required, bit interleaving can be omitted. The bits of one symbol are not spread but combined into one (or a few) trellis segments.

Second, the elimination of the bit interleaver directly leads to the concept of trellis coded modulation (TCM) [14]. Here, since the least significant bits are encoded/decoded jointly, averaging over the coding levels is inherent and no highly unreliable bits occur.

C. Numerical Results

In Fig. 4 numerical results (bit error rate over $E_b/N_0$ in dB) are shown. The channel parameters are the same as above. Bit-interleaved coded modulation with rate-1/2, 64-state convolutional code is used; the transmission rate is 2 bits per dimension (16-QAM uncoded; 256-QAM coded transmission). For MIMO DFE/precoding, no bit interleaving is performed whereas in MIMO OFDM with V-BLAST in each carrier a random bit interleaver is present.

The advantage of TX CSI is clearly visible. In MIMO OFDM, random decision order performs worse, an optimized, but same order for all carriers somewhat better, a per carrier optimized one (which needs TX CSI) even better, and using additionally loading (according to [6]) performs best.
Fig. 5. Bit error rate over the ratio of average transmitted energy per information bit $E_b$ and one-sided noise spectral density $N_0$ in dB. Rate-1/2, 64-state code; MC: multichannel coded; SC: single carrier coded; TC: TX CSI required; Load: use of loading; Sorting: opt.; optimized; p/c: per carrier.

However, due to bad coding levels, uncoded but loaded transmission is still superior for error rates above $10^{-5}$. Coding in combination with precoding (which requires full TX CSI) performs best.

In Fig. 5 rate-1/2 and rate-2/3 64-state convolutional codes are compared. As the performance of the schemes become better, the rate-2/3 code offers a larger gain over the rate-1/2 code. At higher SNR, the coding levels have similar capacities; but going to lower SNR, the differences increase and using the rate-1/2 code coding levels with very poor performance appear. As an example, the capacity limits (transmission rate 2 bits per dimension) for 8-ary signaling, rate-2/3 code and 16-ary signaling, rate-1/2 code are almost the same (see Fig. 3) and the capacities of the binary coding levels are 0.436, 0.710, and 0.854 for 8-ary signaling and 0.075, 0.395, 0.687, and 0.843 for 16-ary signaling. Hence, the capacities are very similar but in the latter situation an additional coding level with very small capacity is present. As a consequence, the poor performance of this level will dominate error rate. This effect becomes even more pronounced when using loading. Now, very large constellations with very poor performing coding levels are employed. Using the rate-2/3 code, smaller constellations are present and performance is superior to that when using the rate-1/2 code.

Applying OFDM with SVD in each carrier, the spread of the noise variances of the subchannels becomes much larger. Typically, the convolutional code will not be able to average sufficiently. Moreover, using loading, a large number of subchannels (the poorest ones) will not be used. This, however, leads to very large constellations in the active subchannels and hence to the appearance of coding levels with very low capacities. Consequently, poor performance will result (not shown) for MIMO OFDM/SVD in combination with BICM.

The solution to overcome the above mentioned problem is to use a more elaborated metric calculation in BICM decoding. Contrary to the usual procedure where the bits within one symbol contribute equally to the decoding metric, a weighting according to their coding level capacities should be done, justified by the theory of information combining [10]. This is subject of current work.

REFERENCES


