Signal Processing in Receivers for Communication over MIMO ISI Channels

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Abstract—Digital transmission over dispersive channels which suffer from intersymbol interference and multiuser interference is reviewed. In particular, the digital signal processing required at the receiver for performing combined spatial/temporal decision-feedback equalization is studied and the aims of processing the received signal are discussed. The calculation of the optimum filters by solving a factorization problem and an practical algorithm for its solution are reviewed. Finally, the optimality of DFE with respect to channel capacity is shown.

I. INTRODUCTION

Over the last years, transmission of multiple data streams in parallel, e.g., by using antenna arrays in transmitter and receiver, creating a multiple-input/multiple-output (MIMO) channel, has become very popular. If, additionally, the channels suffer from intersymbol interference (ISI) (MIMO ISI channel), the received signals are affected by both, ISI and multiuser interference (MUI).

In this paper, we review fast digital transmission over channels with ISI and MUI. The emphasis is on spatial/temporal equalization and the respectively required receiver front-end signal processing. In particular, we concentrate on decision-feedback equalization (DFE), where already detected symbols assist equalization. This practical realization of the chain rule of information theory [4] is an optimum equalization strategy for ISI and MUI channels, in principle. However, the receiver front-end signal processing has to shape the end-to-end impulse response seen by the transmitted data suitably.

In Section II, the channel model is explained. Equalization strategies and the demands on the feedforward processing in DFE receivers are discussed in Section III. Key point for the calculation of the required filters is the solution of a factorization problem, which is stated in Section IV. The main idea and principle steps of a easy-to-use factorization algorithm are reviewed. The optimality of DFE with respect to channel capacity is shown in Section V. A brief summary (Section VI) closes the paper.

II. CHANNEL MODEL

We assume a MIMO ISI channel with $N$ transmit and $M \geq N$ receive antennas. Over each antenna a sequence $\langle a_\nu[k] \rangle$ of $T$-spaced data symbols $a_\nu[k]$, $k \in \mathbb{Z}$, $\nu = 1, 2, \ldots, N$, taken from a signal constellation $\mathcal{A}$ (variance $\sigma_a^2 \triangleq \text{E}\{|a_\nu[k]|^2\}$, $\forall \nu$), is transmitted. The data symbols are combined into the vector $a[k] = [a_1[k], \ldots, a_N[k]]^T$. The continuous-time transmit signal of antenna $\nu$, given in the equivalent complex baseband domain, is obtained through filtering the data sequence by the continuous-time transmit filter with transfer function $T \cdot H_T(f) \leftrightarrow T \cdot h_T(t)$, i.e., $s_\nu(t) = T \sum_k a_\nu[k] h_T(t - kT)$.

The MIMO ISI channel is characterized by its continuous-time impulse responses from each transmit to each receive antenna. Throughout the paper we restrict ourselves to linear time-invariant dispersive channels. Generalization to time-variant channels is straightforward. Denoting the impulse response from transmit antenna $\nu$ to receive antenna $\mu$ by $h_{C,\mu,\nu}(t)$, the continuous-time receive signal at antenna $\mu$ of the ISI MIMO channel reads ($\ast$: convolution)

$$y_\mu(t) = \sum_{\nu=1}^N (s_\nu(t) \ast h_{C,\mu,\nu}(t)) + n_\mu(t) \quad (1)$$

$$= \sum_{\nu=1}^N \sum_k a_\nu[k] \cdot T \left( h_T(t) \ast h_{C,\mu,\nu}(t) \right) + n_\mu(t) \quad (\tau = -kT).$$

It is reasonable to assume the additive Gaussian noise $n_\mu(t)$ at the receive antennas (arranged into the vector $n(t)$) to be spatially and temporally white, each with noise power spectral density $N_0$, i.e., $\text{E}\{n(t + \tau)n^H(t)\} = N_0 \delta(\tau)$.

An important result in communication theory due to T. Ericson [6] states that the optimum receive filter for pulse-amplitude modulated transmission over (single-input/single-output) ISI channels consists of the cascade of a matched filter for $H_C(f)H_T(f)$, $T$-spaced sampling, followed by a discrete-time filter. This principle generalizes to MIMO ISI channels, first stated by W. van Etten [7], [8], where now the optimum receiver consists of a matrix of matched filters for $H_C(f)H_T(f)$, where $H_C(f) = [H_{C,m,\nu}(f)]$ is the matrix of channel transfer functions, $T$-spaced sampling, followed by discrete-time processing. By means of the matrix matched filter the different propagation paths are aligned in time and the impulse energies are concentrated in the sampling instances $kT$. This procedure provides sufficient statistics, i.e., there is no loss of information with respect to data estimation although the sampling theorem is usually not satisfied.

Including transmit filters, matched filtering and $T$-spaced sampling at the receiver, as an intermediate step, a discrete-time channel model is set up, cf. Figure 1. The $N \times N$ discrete-time matrix channel $H_o(z)$, evaluated on the unit circle, calculates to

$$H_o(e^{j2\pi fT}) = \sum_{l=-\infty}^{+\infty} H^H_C(f + \frac{l}{T})H_T(f + \frac{l}{T}) \cdot$$

and the autocorrelation matrix of the additive noise $n_\nu[k]$ reads $\text{E}\{n_\nu[k + \kappa]n^H_\nu[k]\} \leftrightarrow \Phi_{n_\nu,n_\nu}(e^{j2\pi fT}) = \sigma_n^2 H_o(e^{j2\pi fT})$, with $\sigma_n^2 = \frac{N_0}{T}$. 

1Notation: $A^T$: transpose of matrix $A$; $A^H$: Hermitian (i.e., conjugate transpose); $A^{-1}$: inverse of the Hermitian transpose of a square matrix $A$. $I$: Identity matrix. $\ast^{-1}$ abbreviates $(\ast^{-1})^\star$. 

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III. Equalization of MIMO ISI Channels

For generation of decision symbols from which estimates of the data symbols can be obtained, the discrete-time part of the receive filter, succeeding the (matrix) matched filter front-end, has to be chosen suitable. Thereby, both intersymbol (temporal) and multiuser (spatial) interference occur and have to be considered in common.

At first glance, a natural choice would be to perform linear equalization, cf. [7]. The transfer function, $H_o(z)$ (the analytic continuation of $H_o(e^{j2\pi ft})$, given on the unit circle, to the whole complex plane), experienced by the data is equalized by a matrix filter $F(z) = H_o^{-1}(z)$. In this case the end-to-end impulse response reads $I \delta[k]$ ($I \delta[k]$: unit pulse sequence), i.e., ISI and MUI are no longer present and simple symbol-by-symbol threshold decisions can be performed to recover data.\footnote{In practice, the impulse response of the matched filters have to be time-shifted for causal realization. In turn, the end-to-end response is shifted, too, by, say $h_0$, samples and the decisions have to be also delayed by $h_0$ samples. For performance evaluation, this time shift is irrelevant. Hence, without loss of generality, we assume non-causal, two-sided continuous-time filters and restrict ourselves to non-delayed decisions.}

However, the additive noise is enhanced due to the use of the matrix matched filter, and power efficiency of this linear equalization strategy usually is very poor. Moreover, in the case of spectral zeros of $\det(H_o(z))$, linear equalization is even impossible.

The optimum equalization strategy would be maximum-likelihood sequence estimation (MLSE) [12], [8]. Unfortunately, MLSE is only feasible for short impulse responses, small number of transmit antennas and small constellation sizes. In usual scenarios, MLSE is much too complex to be implemented. Hence, in this paper we restrict ourselves to symbol-by-symbol detection schemes.

A very attractive equalization strategy is decision-feedback equalization (DFE), where already detected symbols assist in the equalization and detection of subsequent symbols. DFE is the realization of the chain rule of information theory [4] to practical systems and is very successful in the equalization of ISI channels [16], multiuser detection [5] (there known as successive cancellation) and is the basis of the BLAST detection algorithm [13].

The block diagram of DFE is shown in Fig. 2. First, the samples $y_o[k]$ are passed through a (matrix) feedforward filter $F(z)$. This filter, however, does not primarily equalize the signal, but shapes the end-to-end signal transfer function $H(z) = \sum_k H_k z^{-k} \equiv F(z)H_o(z)$. Additionally, the filtered noise $n[k]$ with noise power spectral density $\Phi_{nn}(e^{j2\pi ft}) = F(e^{j2\pi ft})\Phi_{nn,0}(e^{j2\pi ft})F^H(e^{j2\pi ft}) = \sigma_n^2 F(e^{j2\pi ft})H_o(e^{j2\pi ft})F^H(e^{j2\pi ft})$ has to show some desired properties.

Assuming that correct decisions $\hat{a}[k-\kappa]$, $\kappa = 1, 2, \ldots$, on previous data symbols are already available, their contribution to the ISI and MUI can be subtracted out. This is done by the feedback filter $B(z) - I$, which for zero-forcing (ZF) DFE reads $B(z) = H(z)$. (The generalization to minimum mean-squared error (MMSE) DFE is easily possible.)

For a successive processing of the symbols over space and time (“space-time processing”), the feedforward filter should be chosen such that the following demands are met:

**Causality:** The end-to-end response should exhibit spatial and temporal causality, i.e., $H(z)$ has to have only terms for time indices greater or equal to zero. Having detected the data vectors $a[k\kappa]$, $\kappa < k$, interference caused by them can be canceled in the feedback loop. Additionally, at each time instant $k$, symbols $a_m[k\kappa]$ should be detected successively, i.e., the symbols over space and time should be processed in a zig-
zag fashion. For that, symbol $a_n[k]$ may only interfere into the decision variable of symbols $a_{\mu}[k], \mu > n$. Hence, the zeroth coefficient of the end-to-end response, $H_0$, has to be to be lower triangular. Finally, since suited scaling of the parallel data streams at the receiver is desired, $H_0$ should have a unit main diagonal.

Noise Whitening: Since symbol-by-symbol decisions are aspired, the noise samples at the output of the feedforward matrix filter have to be spatially and temporally uncorrelated. If correlations were present, the reliability of the decision could be improved by prediction of the current noise sample based on prior noise estimates (noise prediction principle, cf., e.g., [10]). This effect is equivalently achieved by a feedforward filter which acts as a noise whitening filter. Hence, for optimality the noise correlation matrix, evaluated on the unit circle, should be diagonal, i.e., $\Phi_{nn}(e^{j2\pi fT}) = \text{diag}(\sigma^2_n, \ldots, \sigma^2_N)$.

Minimum Phase Property and Detection Order: The latter demand gives a constraint on the magnitude of the receive filter; the phase can still be chosen suitably. Since the decisions are based on the diagonal elements of the zeroth coefficient of the end-to-end impulse response and all other terms are canceled, these elements should be as large as possible. It can be shown, that optimum performance is achieved if $H(z)$ is minimum phase, which for matrices is defined as $\det(H(z))$ to have roots only inside the unit circle. Moreover, as the last degree of freedom, the detection ordering of the symbols has to be selected suitably. This ordering can be incorporated into the channel matrix by virtually relabeling the transmit antennas, and hence permuting its columns. A permutation matrix $P$ (non-singular matrix where each row and column contains a single one) can be introduced, which is chosen such that transmission over $H_c(f)P$ and detection in the natural ordering 1 through $N$ gives the best performance.

IV. SPECTRAL FACTORIZATION

For the above-mentioned demands, the optimum discrete-time feedforward matrix filter $F(z)$ (and hence feedback matrix filter $B(z)$) can be calculated given $H_c(e^{j2\pi fT})$ or its analytic continuation $H_c(z)$. This is done by performing a (spectral) factorization, which decomposes $H_c(z)$ according to [8], [17], [10]

$$H_c(z) = S^H(z^{-*}) \cdot \Sigma \cdot S(z).$$

(3)

Here, $\Sigma = \text{diag}(\varsigma_1, \ldots, \varsigma_N)$ is real diagonal, and the matrix polynomial $S(z) = \sum_{k \geq 0} S_k z^{-k}$ is expected to be causal and minimum-phase ($\det(S(z)) \neq 0, |z| \geq 1$). Moreover, $S_0$ has to be lower triangular with unit main diagonal (“monic” matrix polynomial $S(z)$). With this additional constraint, the factorization (3) is unique and it exists as long as $\log(\det(H_c(e^{j2\pi fT})))$ is absolutely integrable [18].

From $S(z)$ and $\Sigma$, the feedforward matrix filter calculates as

$$F(z) = \Sigma^{-1} \cdot S^{-H}(z^{-*}),$$

(4)

which gives the requested causal, minimum-phase, and monic end-to-end transfer function

$$H(z) = \Sigma^{-1} S^{-H}(z^{-*}) \cdot S^H(z^{-*}) \Sigma S(z) = S(z).$$

(5)

The additive noise $n[k]$ is spatially/temporally white with correlation matrix

$$\Phi_{nn}(z) = \sigma^2_n S^{-1} = \sigma^2_n \cdot \text{diag}(\varsigma_1^{-1}, \ldots, \varsigma_N^{-1}).$$

(6)

Hence, neglecting error propagation in the DFE loop, the individual signal-to-noise ratios (SNR) at the input of the decision devices read

$$\text{SNR}_i = \frac{\sigma^2_n}{\sigma^2_i} \varsigma_i, \quad i = 1, \ldots, N.$$  

(7)

In literature, a number of factorization algorithms for solving (3) are discussed, see, e.g., [14]. Among them, the one proposed in [18] seems to be of particular interest. This algorithm is an extension of the polynomial factorization via (repeated) Cholesky decomposition of a coefficient Toeplitz matrix introduced by F.L. Bauer in [2]. He also showed, that the required Cholesky factorization can be done successively, converging to the desired result.

A easy-to-use iterative algorithm for the matrix factorization problem at hand can be derived, if the dual factorization problem, i.e., $M(z) = B(z)B^H(z^{-*})$, has to be solved (which differs from (3) because of the noncommutativity of matrix multiplication). Given $H_c(z)$ we first calculate $B(z) = \sum_{k \geq 0} B_k z^{-k}$, with $B_0$ lower triangular and $\det(B(z)) \neq 0$, such that

$$J H^H_c(z^*) J \equiv M(z) = B(z) \cdot B^H(z^{-*}).$$

(8)

Here, $J = [j_{l,m}], j_{l,N+1-l} = 1$, zero else, is the anti-diagonal identity matrix. The polynomial $M(z) = \sum_{k=0} B_k z^{-k}$ has $N \times N$ coefficient matrices $M^H_k = M^H_k$, which are related to that of $H_c(z)$ (denoted as $H_{o,k}$) by $M_k = J H^H_{o,k} J$.

Similar to [2], (8) can be solved in an iterative way (a lower triangular matrix $\sqrt{X}$ denotes the Cholesky factor of $X$ [3]; $\sum k \leq 0$), cf. also [14]:

$$L_{0,0} = \sqrt{M_0}, \quad m = 1, 2, \ldots$$

(9a)

$$L_{m,i} = \left( M_{m-i} - \sum_{k=0}^{i-1} L_{m,k} L^H_{i,k} \right) L^{-H}_{i,i}, \quad i = 0, \ldots, m - 1$$

(9b)

$$L_{m,m} = \sqrt{M_0 - \sum_{k=0}^{m-1} L_{m,k} L^H_{m,k}}, \quad m \geq 1$$

(9c)

The coefficient matrices $B_k$ of the requested polynomial $B(z)$ are obtained as $B_k = \lim_{m \to \infty} L_{m-k,m}, k = 0, 1, \ldots, m$ [18]. When convergence is reached, i.e., $L_{m,k} \approx L_{m-k,1}, \ldots$, the iteration is stopped.

Finally, having performed the factorization of the above modified problem, $\tilde{S}(z) = [\sum k \tilde{s}_{k,l,m} z^{-k}] \equiv J B^H(z^*) J$ and $D \equiv \text{diag}(\tilde{s}_{0,1}, \ldots, \tilde{s}_{N,N,N})$ are calculated. From these intermediate matrices, the desired ones for the factorization problem (3) are obtained as

$$\Sigma = D^H J \cdot \tilde{S}$$

(10a)

$$S(z) = D^{-1} \tilde{S}(z).$$

(10b)
V. CAPACITY

We now show that assuming high signal-to-noise ratios ($\sigma_n^2/\sigma^2 \gg 1$) and ignoring the effects of error propagation, DFE is an optimum equalization strategy (cf. also [16], [9]). In this region, the optimal power distribution over the parallel channels induced by DFE is a flat one. Moreover, for high SNR, the detection order is of almost no importance and can be chosen arbitrarily. Then, the sum capacity (bits per channel use) of the $N$ parallel channels with SNR given in (7) reads

$$C_{\text{DFE}} = \sum_{i=1}^{N} \log_2 \left(1 + \frac{\sigma_i^2}{\sigma_n^2} \right)$$

$$\approx \log_2 \left( \prod_{i=1}^{N} \frac{\sigma_i^2}{\sigma_n^2} \right)$$

$$= N \cdot \log_2 \left( \frac{\sigma_n^2}{\sigma_n^2} \right) + \log_2 \left( \det (\Sigma) \right), \quad (11)$$

where we have used $\det (\Sigma) = \prod_{i=1}^{N} \sigma_i$, since $\Sigma$ is diagonal.

This capacity is to be compared to that of the underlying MIMO channel. For that we assume that the transmit spectrum is essentially limited to the frequency band $[-1/(2T), 1/(2T)]$, i.e., $H_T(f)$ has brick-wall shape. Within this frequency range a MIMO channel with transfer matrix $H C(f) H_T(f)$, additive white Gaussian noise and SNR $\sigma_n^2/\sigma_n^2$ is present. Its capacity (bits per channel use), using (2) calculates to (cf. [13], [12])

$$C_{\text{MIMO}} = T \int \log_2 \left( \det \left( I + \frac{\sigma_n^2}{\sigma_n^2} H^T_T(f) H C(f) \right) \right) df$$

$$= T \int \log_2 \left( \frac{\sigma_n^2}{\sigma_n^2} H_o(e^{j2\pi f T}) \right) df$$

$$= N \cdot \log_2 \left( \frac{\sigma_n^2}{\sigma_n^2} \right) + T \int \log_2 \left( \det (H_o(e^{j2\pi f T})) \right) df. \quad (12)$$

The remaining integral can be solved using (3) and taking into account that $\det (S(z)) (\det (S^H(z^{-1})))$ is a monic, causal (anti) causal and minimum (maximum) phase polynomial. The corresponding cepstrum is strictly causal (anticausal) [15], hence the coefficient at time index zero, which is given by $T \int \log_2 \left( \det (S(e^{j2\pi f T})) \right) df$ vanishes (as it does for $\det (S^H(z^{-1})))$. We arrive at (cf. also [18])

$$T \int \log_2 \left( \det (H_o(e^{j2\pi f T})) \right) df$$

$$= T \int \log_2 \left( \det (S^H(e^{j2\pi f T}) \Sigma S(e^{j2\pi f T})) \right) df$$

$$= \log_2 \left( \det (\Sigma) \right). \quad (13)$$

In summary we have

$$C_{\text{MIMO}} = N \cdot \log_2 \left( \frac{\sigma_n^2}{\sigma_n^2} \right) + \log_2 \left( \det (\Sigma) \right). \quad (14)$$

which compares to the capacity achievable by DFE, cf. (11). Hence, comparing (11) and (14), we have shown that asymptotically

$$C_{\text{DFE}} = C_{\text{MIMO}} \quad (15)$$

VI. CONCLUSIONS

Digital transmission over channels with ISI and MUI has been reviewed. It has been shown that DFE is well suited for combined spatial/temporal equalization of MIMO ISI channels. The required receiver front-end signal processing has been reviewed and it has been proven that using DFE the capacity of the underlying MIMO ISI channel can be utilized asymptotically. Using precoding schemes [10] this results is not purely theoretical. Combined channel coding and space-time precoding is an attractive strategy for future MIMO transmission systems.

REFERENCES