Single- and Multicarrier Transmission in Multipoint-to-Point Scenarios with Intersymbol Interference: A Comparison

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Abstract

Equalization schemes for multipoint-to-point scenarios (multiple access problem) where the transmission paths exhibit intersymbol interference are investigated. In particular, singlecarrier modulation employing decision-feedback equalization, multicarrier modulation employing BLAST per carrier, and frequency-division multiplexing in disjoint frequency bands are considered. The rate regions, i.e., the region of admissible rates of the users, for these approaches are given and compared. It is shown that despite single- and multicarrier transmission can achieve the same sum rate, in each case MIMO OFDM offers the largest rate region. The results are illustrated via numerical examples.

1 Introduction and Channel Model

In this paper we compare different approaches for transmission in a multipoint-to-point scenario, i.e., multiple access channel (MAC), where the individual transmission paths can not be modeled by flat-fading channels but exhibit intersymbol interference (ISI). Fig. 1 sketches the situation.

![Fig. 1. Multipoint-to-point transmission scenarios over intersymbol interference channels.](image)

Each of the $K$ users—all equipped with one transmit antenna—wants to communicate a sequence $(a_{\mu}[k])$ of $T$-spaced data symbols $a_{\mu}[k]$, $k \in \mathbb{Z}$, $\mu = 1, 2, \ldots, K$, taken from a signal constellation $\mathcal{A}$ (variance $\sigma_a^2 \triangleq \mathbb{E}[(a_{\mu}[k])^2], \forall \mu$), to the central office which has $N_R \geq K$ receive antennas. We combine the data symbols into the vector $\mathbf{a}[k] = [a_1[k], \ldots, a_K[k]]^T$.

The $T$-spaced discrete-time channel models between each pair of transmit and receive antenna (given in the equivalent complex baseband domain [8] and comprising pulse shaping at the transmitter, the actual continuous-time channel, matrix matched filtering, and $T$-spaced sampling) are assumed to be (at most) of degree $L-1$ ($L$: length of the impulse responses) and denoted by $H_{\nu\mu}(z), \mu = 1, \ldots, K, \nu = 1, \ldots, N_R$, where the column index $\mu$ corresponds to the user and the row index $\nu$ to the receive antenna. The transfer functions are preferably combined into the channel matrix polynomial $\mathbf{H}(z) = \{H_{\nu\mu}(z)\}$. The additive discrete-time Gaussian noise is assumed to be spatially and temporally white with variance $\sigma^2_n = N_0/T$.

Since in addition to multiuser/multicarrier interference (MUI/ISI) ISI is present, equalization/suppression of both types of interference is required. In particular we consider (i) singlecarrier (SC) transmission using decision-feedback equalization (DFE), jointly for MUI/ISI, (ii) multicarrier (MC) transmission using orthogonal frequency-division multiplexing (OFDM) for dealing with the ISI and DFE per carrier (BLAST) for treating the MUI, and (iii) frequency-division multiplexing (FDM), where the users operate in disjoint frequency bands. The transmission/equalization schemes are compared by studying their rate regions, i.e., the region of achievable rates of the users.

Noteworthy, numerous prior work on the comparison of single- and multicarrier techniques is available in literature. However, either single-input/single-output (SISO) or point-to-point multiple-input/multiple-output (MIMO) scenarios are studied, cf. [16], [7], [2] and the references therein. In the present work, the comparison is done for the ISI MAC problem.

In Section 2, single- and multicarrier transmission techniques are briefly reviewed and their performance calculation is given. Section 3 presents numerical examples for the two and three user case and discusses the similarities and differences between the schemes under consideration. We focus on the situation where the users have no or only very limited amount of channel state information (CSI) available (in all cases perfect CSI is assumed at the receiver). Hence, waterfilling is only briefly mentioned; the power spectral density (PSD) of the transmit signal is expected to be flat. Concluding remarks are made in Section 4.
2 Transmission Techniques

2.1 Singlecarrier Modulation

2.1.1 Structure and Signal-to-Noise Ratio

First, MUI and ISI can be treated jointly by performing DFE with respect to users and time. DFE is also known as successive cancellation in multiuser systems. In order to enable successive detection and cancellation, the received signal is passed through a matrix feedforward filter \( F(z) \), which shapes the end-to-end signal transfer function \( C(z) = \sum_k C_k z^{-k} \equiv F(z)H(z) \), see Fig. 2.

![Diagram](image)

Fig. 2. Transmission system with combined MUI/ISI decision-feedback equalization. For flat-fading MIMO channel the time index \( k \) is dropped and the matrix filters reduce to constant matrices.

In zero-forcing (ZF) DFE, the feedback filter \( B(z) \) is identical to the end-to-end response \( C(z) \). Here, the feedforward filter should be chosen such that \( C(z) \) exhibits spatial and temporal causality. Moreover, the noise samples at the output of \( F(z) \) have to be spatially and temporally white. Optimizing the DFE according to the minimum mean-squared error (MMSE) criterion leads to a trade-off between noise enhancement and residual MUI/ISI. Here, in contrast to the ZF approach, end-to-end signal transfer function \( C(z) \) and feedback matrix filter \( B(z) \) differ (in \( C(z) \) an anti-causal part is present which reflects precursors which are not forced to zero here), see, e.g., [6].

The required matrix feedforward and feedback filters are obtained by performing a spectral factorization, which decomposes \( H_o(z) \equiv H^H(z^{-*})H(z) \) according to [6]

\[
P^T \left( H_o(z) + \zeta I \right) P = B^H(z^{-*}) \Sigma \cdot B(z) .
\]

Here, \( \Sigma = \text{diag}(\sigma_1^2, \ldots, \sigma_K^2) \) is a real diagonal matrix, \( P \) a permutation matrix, and the matrix polynomial \( B(z) = \sum_{k \geq 0} B_k z^{-k} \) (which immediately gives the feedback filter) is expected to be causal and minimum-phase. Moreover, \( B_0 \) has to be lower triangular with unit main diagonal (“monic” matrix polynomial \( B(z) \)). For the ZF solution, \( \zeta = 0 \) is selected, whereas in the MMSE setting \( \zeta = \frac{\sigma_n^2}{\sigma_o^2} \). The feedforward matrix filter then calculates to

\[
F(z) = \Sigma^{-1} B^{-H}(z^{-*})P^T H^H(z^{-*}).
\]

The additive Gaussian noise is white (with respect to users and time) with correlation matrix \( \sigma_n^2 \Sigma^{-1} = \sigma_o^2 \text{diag}(\zeta_1^{-1}, \ldots, \zeta_K^{-1}) \). Compensation of the unavoidable bias in the MMSE solution [1], [6] and neglecting error propagation in the feedback loop, DFE decomposes the MIMO channel into decoupled, parallel channels of the users, whose individual signal-to-noise ratios at the input of the decision devices read

\[
\text{SNR}_\mu = \frac{\sigma_o^2}{\sigma_n^2} \zeta_\mu - 1 , \quad \mu = 1, \ldots, K .
\]

In the ZF case, the “-1” in the above equation is not present.

2.1.2 Rate Region

Having the SNRs, assuming Gaussian transmit symbols, and ignoring the fact that DFE requires immediate decisions, \(^3\) the achievable rates of the users for the fixed permutation \( P \) are given by

\[
R_{\mu} = \log_2 \left( 1 + \text{SNR}_\mu \right) , \quad \mu = 1, \ldots, K
\]

\[
= \begin{cases} 
\log_2 \left( 1 + \frac{\sigma_o^2}{\sigma_n^2} \zeta_\mu \right) , & \text{ZF} \\
\log_2 \left( \frac{\sigma_o^2}{\sigma_n^2} \zeta_\mu \right) , & \text{MMSE}
\end{cases}
\]

(4)

Each given permutation \( P \) leads to a tuple \( R \equiv [R_1, \ldots, R_K] \) of maximum rates of the users. Since lower rates are also feasible, the rate tuple defines a rate region, a hyper cuboid whose extreme vertices are \( R \) and \( 0 \). Via time-sharing arguments [3], the rate region using DFE (SC technique) is then given as the convex hull over the rate regions corresponding to the rate tuples \( R \) for all possible permutations \( P \), formally we write

\[
\mathcal{R}^{(SC)} = \text{Convex Hull} \{R_{P_1}, \ldots, R_{P_K}\} .
\]

(5)

Using (4), the sum rate in singlecarrier schemes is given by (MMSE case)

\[
R_{\text{sum}}^{(SC)} = \log_2 \left( \prod_{\mu=1}^{K} \zeta_\mu \right) .
\]

(6)

Considering the spectral factorization (1), the product over the elements \( \zeta_\mu \) is given by, cf. [7],

\[
\log_2 \prod_{\mu=1}^{K} \zeta_\mu = \int_{0}^{1} \log_2 \det \left( H_o(e^{i2\pi\phi}) + \frac{\sigma_n^2}{\sigma_o^2} I \right) d\phi .
\]

(7)

Combining both equations and taking the definition of \( H_o(z) \) into account, the sum rate for MMSE DFE is hence given as

\[
R_{\text{sum}}^{(SC)} = \int_{0}^{1} \log_2 \det \left( I + \frac{\sigma_n^2}{\sigma_o^2} H_o(e^{i2\pi\phi}) H^H(e^{-i2\pi\phi}) \right) d\phi .
\]

(8)

\(^3\)This problem may be solved by means of interleaving, cf. [4].

\(^4\)When glimpsing at (4), one may wonder that the ZF approach might lead to a larger rate than the MMSE approach. This is not the case—the unbiased MMSE SNR is always larger (or equal) to the ZF SNR, i.e., \( \zeta_\mu - \zeta \) in the MMSE case is larger than \( \zeta_\mu \) in the ZF case.
Noteworthy, this rate is the same as in point-to-point communications over a MIMO channel using constant transmit PSD.

### 2.2 Multicarrier Modulation

#### 2.2.1 Structure and Signal-to-Noise Ratio

Next, MUI and ISI may be equalized separately. Temporal interference is treated by using multricarrier modulation; in the present situation each user applies an OFDM transmitter, and at each receive antenna an OFDM receiver is present.

Assuming a guard interval of sufficient length, the MIMO ISI channel is decomposed into a set of, say $D$, parallel, independent flat fading MIMO channels. The channel matrix of the MIMO channel at carrier $d$ is related to the transfer function of the underlying channel by

$$
H^{(d)} = H(\epsilon^{2\pi(d-1)/D}), \quad d = 1, \ldots, D .
$$

The additive noise remains white with correlation matrix $\Sigma = \sigma_n^2 I$.

In the second step, in each of this parallel channels (the carriers), user detection is performed individually. Here we assume again the application of DFE (successive cancellation), in the context of (flat-fading) MIMO channels also known as V-BLAST. The required matrices are obtained by calculating a sorted (permutation matrix $P$) Cholesky factorization per carrier

$$
(P^{(d)})^T (H^{(d)} + \zeta I) P^{(d)} = (B^{(d)})^H \Sigma^{(d)} B^{(d)}
$$

with Hermitian matrix $H^{(d)} = (H^{(d)})^H$ and real diagonal matrix $\Sigma^{(d)} = \text{diag}(\varsigma_d, \ldots, \varsigma_K)$. Again, for the ZF solution, $\zeta = 0$ is selected, whereas in the MMSE setting $\zeta = \frac{\sigma_n^2}{\sigma_a^2}$. The lower triangular matrix with unit main diagonal $B^{(d)}$ immediately gives the feedback matrix, whereas the feedforward matrix calculates to

$$
F^{(d)} = (\Sigma^{(d)})^{-1} (B^{(d)})^{-H} (P^{(d)})^T (H^{(d)})^H .
$$

Using this MIMO OFDM approach and ignoring error propagation in the DFE, in each of the $D$ carriers, $K$ parallel, independent channels with signal-to-noise ratio

$$
\text{SNR}_d^{(\mu)} = \frac{\sigma_a^2}{\sigma_n^2} \varsigma^{(\mu)}_d - 1, \quad \mu = 1, \ldots, K, \quad d = 1, \ldots, D ,
$$

are present. As above, in the ZF case, the “-1” (resulting from bias compensation) in the SNR calculation does not appear.

#### 2.2.2 Rate Region

From the SNRs and assuming Gaussian transmit symbols, the achievable rates of the users for given permutations $P^{(d)}$ can be calculated. Ignoring the loss due to the guard interval and assuming a constant PSD of the users transmit signals, we have

$$
R_\mu = \frac{1}{D} \sum_{d=1}^D \log_2 \left( 1 + \text{SNR}_d^{(\mu)} \right), \quad \mu = 1, \ldots, K
$$

$$
= \left\{ \begin{array}{ll}
\frac{1}{D} \sum_{d=1}^D \log_2 \left( 1 + \frac{\sigma_n^2}{\sigma_a^2} \varsigma^{(\mu)}_d \right), & \text{ZF} \\
\frac{1}{D} \sum_{d=1}^D \log_2 \left( \frac{\sigma_a^2}{\sigma_n^2} \varsigma^{(\mu)}_d \right), & \text{MMSE}
\end{array} \right. \quad (13)
$$

Since the users cannot cooperate and coding for achieving capacity has to be done across the carriers, only global sorting and no individual permutation per carrier is possible, i.e., $P^{(d)} = P, \forall d$. Then, the rate region using MIMO OFDM (MC technique) is given as the convex hull over regions corresponding to the rate tuples $R = \{R_1, \ldots, R_K\}$ for all $K$ possible permutations $P$

$$
\mathcal{R}^{(MC)} = \text{Convex Hull} \{ R_{P_1}, \ldots, R_{P_K} \} . \quad (14)
$$

Using MMSE DFE per carrier, the sum rate per channel use is given by (cf. (13))

$$
R_{\text{sum}}^{(MC)} = \frac{1}{D} \sum_{d=1}^D \log_2 \left( \prod_{\mu=1}^K \frac{\sigma_n^2}{\sigma_a^2} \varsigma^{(\mu)}_d \right) , \quad (15)
$$

and from the factorization (10), we have (see [7])

$$
\prod_{\mu=1}^K \varsigma^{(\mu)}_d = \det (H^{(d)} + \sigma_a^2 I) . \quad (16)
$$

This leads to (using the definition of $H^{(d)}$)

$$
R_{\text{sum}}^{(MC)} = \frac{1}{D} \sum_{d=1}^D \log_2 \det \left( I + \frac{\sigma_n^2}{\sigma_a^2} (H^{(d)})^H H^{(d)} \right) , \quad (17)
$$

which tends to (8) as $D \rightarrow \infty$, i.e., a sufficiently large number of carriers is present. Hence both approaches (SC and MC) may achieve the same sum rate. Please note that both results do not depend on the permutation $P$, as it vanishes in (7) and (16) since $\det(P) = \pm 1$.

Moreover, performing (10) for the 2 user case (dropping the superscript) and given permutation (here user 1, followed by user 2), we immediately have ($\zeta = \frac{\sigma_n^2}{\sigma_a^2}$)

$$
H_0 + \varsigma I \overset{\text{def}}{=} \left[ \begin{array}{cc}
A + \varsigma & B \\
C + \varsigma & D
\end{array} \right] = B^H \Sigma B . \quad (18)
$$

with $\Sigma = \left[ \begin{array}{cc}
A + \varsigma & -\frac{1}{\sqrt{2}} \\
0 & C + \varsigma
\end{array} \right]$ and $B = \left[ \begin{array}{c}
1 \\
0
\end{array} \right]$. The unbiased SNRs thus read $\text{SNR}_1 = \frac{\sigma_n^2}{\sigma_a^2} (A + \varsigma - |B|^2 (C + \varsigma)) - 1$ and $\text{SNR}_2 = \frac{\sigma_n^2}{\sigma_a^2} (C + \varsigma) - 1 = \frac{\sigma_n^2}{\sigma_a^2} C$. By inspection it can be seen, that $\text{SNR}_1$ is larger in the MMSE case than in the ZF case ($\varsigma = 0$), whereas $\text{SNR}_2$ is the same for both settings. Hence, only the user detected first gains due to the MMSE approach; the user detected last does not gain. In the $K$ user case, the first $K - 1$ users gain due to the MMSE solution, whereas the last user has the same SNR as for the ZF solution. Due to the assumption of perfect feedback (perfect interference cancellation), in each case the last user “sees” a interference-free (clean) channel, and no further improvement is possible.
2.2.3 Comparison

Considering the definition of $H_{\nu}^{[d]}$, the $K^{th}$ element of $\Sigma$ (corresponding to the user detected last) reads
\[\sum_{\nu=1}^{\nu_d} |H_{\nu}^{[d]}(e^{2\pi j(d-1)/D})|^2 + \zeta, \ d = 1, \ldots, D.\]
The (unbiased) spectral SNR of this user hence reads
\[
\text{SNR}_K(a^{2\pi f \phi}) = \frac{\sigma_n^2}{\sigma_a^2} \sum_{\nu=1}^{N_r} |H_{\nu}^{[a]}(e^{2\pi f \phi})|^2.
\]

It is worth noting that this is the spectral SNR which is achievable when performing maximum-patio combining (MRC) in each frequency bin (weighting with $h_{\mu}^{[\nu]}(e^{2\pi f \phi})$ at the receiver, where $h_{\mu}(z)$ denotes the $\mu^{th}$ column of the channel matrix $H(z)$). Since MRC leads to the maximum SNR among all receiving strategies it also leads to the maximum rate. All other receive filters—in particular the feedforward matrix filter $F(z)$ in MUI/ISI DFE (SC scheme)—result in lower SNRs and rates. Hence, the bounds on the maximum individual rates of the users are lower in the SC scheme compared to the MC scheme: the rate region of MC schemes are larger than that of SC schemes. Only for orthogonal channels ($H^{[a]}(z^{-1})H(z)$ is a diagonal matrix), single- and multicarrier techniques lead to the same, in that case, rectangular rate region.

2.3 Frequency-Division Multiplexing

Besides the above mentioned MIMO techniques, FDM can be used as accessing scheme, cf. [12]. The users transmit in different frequency bands and hence cause no MUI. Each receiver only has to handle its own ISI.

Having $K$ users, the entire frequency band (normalized frequency $\phi \in [0, 1]$) is divided into $K$ disjoint bands $B_{\mu}$, i.e., $B_{\mu} \cap B_{\nu} = \{\}$, $\forall \mu, \nu, \mu \neq \nu$ and $\bigcup_{\nu=1}^{K} B_{\nu} = [0, 1]$.

The achievable rate of the users (assuming Gaussian signaling) is then given as
\[
R_{\mu} = \int_{B_{\mu}} \log_2 \left(1 + \gamma \frac{\sigma_n^2}{\sigma_a^2} h_{\mu}^{[\nu]}(e^{2\pi f \phi})h_{\mu}(e^{2\pi f \phi})\right) d\phi
\]
\[
\text{(20)}
\]

The factor $\gamma$ corresponds to two possible power allocation strategies: if a constant PSD is desired (limitations due to spectral masks), $\gamma = 1$ and the individual powers are smaller compared to the above strategies. If fixed power per user is desired, this power is concentrated within the usable frequency band $B_{\mu}$ and hence the PSD is increased by the factor $\gamma = 1/|B_{\mu}|$, where $|\cdot|$ denotes the measure of the frequency band, i.e., the (normalized) bandwidth. The rate region achievable using FDM is then given as the convex hull over the rate tuples for all possible frequency allocations.

![Fig. 3](image-url) Fig. 3. Illustration of partitioning of the frequency band in the 2 user case. Dashed: example of magnitude of channel transfer function.

Fig. 3 shows the two feasible partitionings of the frequency band in the 2 user case. Due to the different channel transfer functions, both possibilities may lead to different results (in contrast to time sharing in static channels where the time-sharing pattern is irrelevant).

3 Numerical Examples

In this section, the rate regions corresponding to the above transmission schemes are visualized by means of numerical examples. We concentrate the discussion on the 2 user case and assume $N_R = 2$ receive antennas; in particular we employ a $2 \times 2$ MIMO ISI channel model with 3 taps, which reflects a typical equal-gain scenario. In order to emphasize the differences, the SNR is fixed to $10 \log_{10}(\sigma_n^2/\sigma_a^2) = 0$ dB. In each case, perfect feedback in the DFE is assumed and the guard interval in OFDM schemes using $D = 256$ carriers is ignored. Moreover, constant PSD of the transmit signals is assumed—in contrast to waterfilling, here no CSI at the transmitter is required.

3.1 Singlecarrier Modulation

Fig. 4 (left) sketches the rate regions for singlecarrier modulation using ZF and MMSE DFE. The point marked by a square corresponds to first detecting user 1 and then user 2, whereas the point marked by a circle gives the situation when first user 2 and then user 1 is detected. It is clearly visible that using the MMSE approach, both users gain; the rate region using the ZFfrontend is completely included within the region employing the MMSE frontend. Moreover, for the ZF approach, the sum rate differs for both detection orders, whereas in case of the MMSE approach the sum rate is independent of the detection order, as shown above.

3.2 Multicarrier Modulation

The middle part of Fig. 4 sketches the rate regions for multicarrier modulation using ZF and MMSE DFE in each carrier. The vertices of the rate pentagon again correspond to the situation where user 1 is first detected and then user 2 (same ordering in all carriers) and vice versa. It can be observed that using the MMSE approach, only the user detected first gains (see above).

3.3 Frequency-Division Multiplexing

Finally, on the right hand side of Fig. 4, the rate regions using FDM are plotted. The light curves give the boundaries of the rate regions for both possible arrangements of the 2 users in the frequency band. The points of the curves are obtained by varying the percentage utilized by users 1 and 2. The rate region using FDM (heavy lines) is given as the convex hull over the union of these regions. Two situations are

\[5\text{For the numerical results, } H(z) = \begin{bmatrix}
-55+54j & 16-75j \\
20-58j & 21+124j \\
-30-46j & -39-110j \\
-23-23j & 39-78j
\end{bmatrix} z^{-1} + \begin{bmatrix}
-79-91j \\
29-66j \\
-38+14j \\
-11-05j
\end{bmatrix} z^{-2} \text{ is used.}\]
plotted: fixed PSD, hence the total power per user varies (dashed); and fixed transmit power of the users, hence the PSD varies. It is obvious that when using fixed transmit powers in contrast to fixed PSD, higher sum rates and a larger rate region are possible.

3.4 Discussion

For comparison, the relevant rate regions are repeated on the left hand side of Fig. 5. The observations from this figure are as follows: First, single- and multicarrier modulation (both employing MMSE filters) approach the same sum rate, cf. (8), (17). However, multicarrier modulation exhibits a larger rate region, since the user detected last in the MC scheme exhibits a interference-modulation exhibits a larger rate region, since the user the same sum rate, cf. (8), (17). However, multicarrier modulation (both employing MMSE equalization. Square: decision order 1, 2; circle: decision order 2, 1. SNR: 10 \log_{10}(\sigma^2_a/\sigma^2_n) = 0 \text{ dB}. K = N_R = 2.

Fig. 5. Rate regions for singlecarrier transmission (dash dotted), multicarrier transmission (solid), and frequency-division multiplexing (dashed). MMSE equalization. Square: decisions order 1, 2; circle: decision order 2, 1. Variation of SNR. K = N_R = 2.

The rate region for FDM (constant transmit power of the users) coincides with that of multicarrier in the edge points \((R_{1,\text{max}}, 0)\) and \((0, R_{2,\text{max}})\), where in both situations one user utilizes the entire frequency band and the second is not present. The rate region of FDM may exceed that of MIMO OFDM (visible in Fig. 5, left). This effect occurs since in contrast to MIMO OFDM, FDM has at least some CSI available at the transmitter (the frequency band and hence the power allocation). However, in the present MIMO case, the FDM rate region is not guaranteed to touch the dominant face of the multicarrier rate region (as it is the case in time- or frequency sharing in (flat) Gaussian multiple access channels, see, e.g., [3, Page 406]). Typically, in the present situation, FDM offers only a lower sum rate compared to single-/multicarrier modulation.

Fig. 5 compares the rate regions for different SNRs. The above explained phenomena are visible in all situations. Going to larger SNRs, the differences between single- and multicarrier techniques become smaller and the rate regions approach each other. Using FDM in disjoint frequency bands is especially not rewarding in the high SNR region; here the boundary of the rate region approaches a straight line. This is because the sum rates of FDM and MC/SC differ in a factor of \(K\), the multiplexing gain of the MIMO channel.

The effect of non-constant PSD is assessed in Fig. 6, where the rate regions for constant PSD are compared with that obtained when performing waterfilling (individually per user). Both FDM and MIMO OFDM with MMSE DFE per carrier gain from an optimized transmit power allocation. Now, the FDM rate region is entirely contained in the region for multicarrier modulation. Noteworthy, since both users gain differently from waterfilling, the sum rate depends on the detection order (as in the ZF case).

A comparison of the rate regions of SC, MC, and

Fig. 6. Rate regions for multicarrier transmission and frequency-division multiplexing. MMSE equalization. Dashed: constant PSD; Solid: Waterfilling. Square: decision order 1, 2; circle: decision order 2, 1. SNR: 10 \log_{10}(\sigma^2_a/\sigma^2_n) = 0 \text{ dB}.

A comparison of the rate regions of SC, MC, and
FDM schemes for the three user case ($K = N_R = 3$), MMSE equalization and randomly selected but fixed channel matrix is given in Fig. 7. The above derived and explained phenomena are visible in this situation, too. In particular, the rate region of SC is entirely contained in that of MC; their boundaries touch at the dominate face, since SC and MC achieve the same sum rate.

Starting from the above discussion on the capacity regions for a fixed channel realization, statistics on the achievable rate tuples for random channel realizations can be deduced. In turn, predictions on the outage capacity of the respective schemes can be made. In Fig. 8, contour plots of the histograms of rate regions for $K = N_R = 2$, i.e., the outage regions, are plotted. Inside the regions the outage probability of the respective rate pairs is smaller than the labeled percentage. MMSE equalization. SNR: $10 \log_{10}(\sigma_n^2/\sigma_a^2) = 0$ dB. $K = N_R = 3$.

4 Conclusions

Singlecarrier modulation employing spatial/temporal decision-feedback equalization, multicarrier modulation employing BLAST per carrier, and frequency-division multiplexing in disjoint frequency bands are assessed in multipoint-to-point scenarios where the channels exhibit intersymbol interference. It has been shown, that despite single- and multicarrier transmission can achieve the same sum rate, in each case MIMO OFDM offers the largest rate region. Frequency-division multiplexing in disjoint frequency bands is always inferior.

Further work has to consider non-uniform transmit PSD obtained by joint waterfilling over the users, e.g., [14], which, however, requires a higher amount of CSI at the transmitters. Moreover, considering the uplink-downlink duality [10], [11], [15], the present results can be readily transformed to point-to-multipoint scenarios. Due to the construction of the rate region in this broadcast case from that in the multiaccess case, multicarrier modulation will here also give a larger rate region.

References