Selected Mapping with Implicit Transmission of Side Information using Discrete Phase Rotations

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***Abstract*— Selected mapping is a popular scheme for reducing the peak-to-average power ratio in orthogonal frequency-division multiplexing (OFDM) systems. However, when using this scheme, the transmission of side information is necessary. In this paper, we introduce a novel approach to transmit the side information by embedding it into the OFDM frame. By rotating the QAM constellation in selected carriers by $\pi/4$, new degrees of freedom are created. No rate loss and no increase in transmit power is introduced by this approach. Numerical results cover the very good error performance of this scheme, which is almost equal to perfect transmission of the side information.**

I. Introduction

In wireless communication systems the application of multicarrier modulation is popular for digital transmission over frequency-selective channels. In particular, *orthogonal frequency-division multiplexing (OFDM)* [5] is a very popular implementation.

One of the most serious problems of OFDM is the large *peak-to-average power ratio (PAR)* of the transmit signal. The non-linear power amplification of such a signal at the transmitter front-end leads to signal clipping and hence signal distortion, and, even more serious, generation of out-of-band radiation. To avoid the violation of spectral masks the power amplifier has to operate at large power back-offs, which reduces the power efficiency.

In order to increase the power efficiency, an algorithmic control of the peak-power of the signal at the transmitter is required. Over the last years a variety of so-called PAR reduction algorithms has been developed. An overview of the basic ideas can be found in [12].

In this paper, we consider *selected mapping (SLM)* [3], one of the most popular schemes for PAR reduction in literature. However, applying SLM, transmission of some extra information (side information), which is especially prone to transmission errors, is required.

In the following, we introduce and analyze a novel approach to transmit the extra information. Main advantage of this method is that this side information is embedded completely into the OFDM frame completely and no loss in transmission rate and no increase of transmit power occurs.

This paper is organized as follows: Section II introduces the system model. In Section III a short review of SLM is given. Section IV considers the embedding of the side information into the transmitted OFDM frame. The decoding of the side information at the receiver is introduced in Section V. Section VI presents numerical results and Section VII discusses a possible combination of this novel approach with extensions of SLM. Conclusions are drawn in Section VIII.

This work was supported by Deutsche Forschungsgemeinschaft (DFG) within the framework TakeOFDM under grants FI 982/1-1 and 1-2.

II. System Model

Subsequently, we consider single-antenna OFDM transmission. The frequency-domain OFDM frame is given by the vector[1] $\boldsymbol{A} = [A_d]$ with $d = 0, \ldots, D-1$. Each frequency-domain symbol A_d is drawn from an M-ary (square) QAM constellation $\mathcal{A}_M$ with signal points $A_d \in \{\pm A_{\mathrm{I}} \pm \mathrm{j}A_{\mathrm{Q}}\}$ with $A_{\mathrm{I/Q}} \in \{\frac{1}{2}, \frac{3}{2}, \ldots, \frac{\sqrt{M}-1}{2}\}$. The variance of the constellation is hence given by $\sigma_a^2 = \mathrm{E}\{|A_d|^2\} = \frac{M-1}{6}$.

The frequency-domain OFDM frame $\boldsymbol{A}$ is then transformed into time domain by the *inverse discrete Fourier transform (IDFT)*. The resulting time-domain OFDM frame is written as the vector $\boldsymbol{a} = [a_k]$ with $k = 0, \ldots, D-1$, whereby the elements of $\boldsymbol{a}$ are calculated by $a_k = \frac{1}{\sqrt{D}} \sum_{d=0}^{D-1} A_d \mathrm{e}^{\mathrm{j}2\pi kd/D}$. Due to the definition of the Fourier transform, the variance of the time-domain signal is also σ_a^2.

The time-domain signal[2] is then transmitted over an AWGN multi-path channel described by the transfer function $H(z)$ of length l_{H}. The variance of the additive (complex-valued) noise is σ_n^2. At the receiver, after calculation of the *discrete Fourier transform (DFT)*, the (complex-valued) fading coefficients $H_d = H(\mathrm{e}^{\mathrm{j}2\pi d/D})$ in all subcarriers are compensated by means of equalization. Hence, at each subcarrier Gaussian noise with variance $\sigma_n^2/|H_d|^2$ is present.

III. Review of Selected Mapping

In [3], the original approach of SLM has been proposed. The idea of this approach is as follows. Given the original OFDM frame, U different signal candidates are generated via U different bijective mappings. In the original work [3] these mappings are realized via an element-wise multiplication with randomly chosen phase vectors (candidate generating vectors) $\boldsymbol{P}_{\mathrm{c}}^{(u)} = [P_{\mathrm{c},d}^{(u)}]$, $d = 0, \ldots, D-1$, $u = 1, \ldots, U$, with $P_{\mathrm{c},d}^{(u)} \in \{\pm 1, \pm \mathrm{j}\}$. With this discrete choice of phase rotations the signal candidates are also drawn from the original modulation alphabet $\mathcal{A}_M$. Thus the synchronization algorithm at the receiver is identical as without PAR reduction.

Each signal candidate is then transformed into time domain and its PAR value is determined. Then, the "best" signal

[1] Notation: Throughout this paper vectors are denoted with bold letters. Frequency-domain symbols are written as capital letters, time-domain symbols as lower-case letters. $\lceil\cdot\rceil/\lfloor\cdot\rfloor$ symbolizes rounding up/down to the next integer towards plus/minus infinity. $\mathcal{CN}(m, \sigma^2)$ describes a complex Gaussian random process with mean value m and variance σ^2. An element-wise multiplication of two vectors is described by $\odot$.

[2] For convenience, we do not consider appending the cyclic prefix, pulse shaping, and modulation to radio frequency.

representation, i.e., the one exhibiting the lowest PAR (index $\check{u}$) is chosen for transmission. A block diagram of this approach is depicted in the left part of Fig. 1.

In order to obtain the original OFDM frame at the receiver, it is essential to undo the applied mapping on the initial OFDM frame. Hence, the transmission of side information—here the index $\check{u}$—is required. For that, at least $\mu = \lceil \log_2(U) \rceil$ bits have to be spend. Moreover, the side information has to be protected extraordinarily as an erroneous estimation of the index $\check{u}$ will lead to the loss of the entire OFDM frame.

Subsequently, we will introduce and analyze a novel scheme for transmission of the side information.

IV. Embedding of the Side Information

A. Embedding the Side Information into the OFDM Frame

In the following the index $\check{u}$ of the actual chosen signal candidate is expressed as a word of binary symbols written as a vector $\boldsymbol{b} = [b_i]$, $i = 1, \ldots, \mu$, $\mu = \lceil \log_2(U) \rceil$, $b_i \in \{\pm 1\}$. The binary information is now embedded into the (frequency-domain) OFDM frame, whereby each subcarrier represents exactly one bit. This is done in that way, that $b_i = -1$ is represented by the original QAM constellation and $b_i = +1$ by the QAM constellation rotated by $\pi/4$. Now, the receiver has to estimate the combination of rotated subcarriers and can therefore determine the index of the applied mapping on the actual OFDM frame.

In principal, each OFDM frame is able to code the side information with D bits. However, usually the number μ of required bits is much smaller than D. Hence, this degree of freedom can be used to protect the side information against transmission errors. Subsequently, we use a simple $(r\mu, \mu)$ repetition code [16] with r repetitions. However, any block code (e.g., a Hamming code) may be used.

In order that the receiver-sided synchronization algorithm does not get confused by the $\pi/4$ phase rotations, the first position of one coded side information index is never rotated and serves as reference. Now, the synchronization algorithm is able to base its estimation on the subcarriers which are not rotated. Hence, the number of repetitions is limited by $r = \lfloor D/(\mu + 1) \rfloor$. At the remaining subcarriers the original constellation stays. The resulting codeword $\boldsymbol{c}^{(u)}$ (for $\boldsymbol{b} \,\widehat{=}\, u$) of length D reads

$$\boldsymbol{c}^{(u)} = [c_d^{(u)}] = [-1 \quad \boldsymbol{b} \quad -1 \quad \boldsymbol{b} \quad \cdots \quad -1 \quad \cdots]. \qquad (1)$$

The sets of indices, where the i^{th} element of the vector $\boldsymbol{b}$ occurs within $\boldsymbol{c}^{(u)}$, are denoted as $\mathcal{I}_{\text{si},i} = \{i + m(\mu+1) \mid m = 0, \ldots, r-1\}$, $i = 1, \ldots, \mu$.

The embedding of the side information into the OFDM frame can now be described by an element-wise multiplication of the u^{th} signal candidate with phase vector (side information embedding vector)

$$\boldsymbol{P}_{\text{si}}^{(u)} = [\mathrm{e}^{\mathrm{j}\frac{\pi}{8}(c_d^{(u)}+1)}] \qquad d = 0, \ldots, D-1. \qquad (2)$$

If the signal candidates are generated by phase rotations as described in the original approach of SLM the embedding of the side information can be combined with the candidate generating vector to the entire candidate generating vector (see Fig. 1)

$$\boldsymbol{P}^{(u)} = \boldsymbol{P}_{\text{c}}^{(u)} \odot \boldsymbol{P}_{\text{si}}^{(u)}, \quad u = 1, \ldots, U. \qquad (3)$$

Even with the restriction to a special choice of phase vectors, described above, all signal candidates remain statistically independent and the ccdf matches the analytical result from [3, Eqs. (2) and (3)]. Hence, the significant gains of SLM, well known from literature, are also achievable with this approach.

B. Discussion

In literature there exist a wide range of different techniques to transmit the side information. One of the most important techniques is the scrambler variant of SLM [6]. With this technique, the side information is not transmitted explicitly, but is dispersed over the whole OFDM frame. However, with this variant of SLM error propagation within the receiver-sided descrambler occurs.

In [13], [4], [14], [1] schemes have been proposed, which modulate (either by phase or amplitude) the side information within the information carrying signal. In [13], [4], [1] the mappings are not performed with discrete phase vectors but continuously distributed ones. In practical realizations, this scheme is hardly feasible to implement, as continuously distributed phase vectors would disturb the receiver-sided synchronization algorithm. Moreover, the estimation of continuous phase rotations is very complex. Our novel approach bases on the same idea. However, we avoid synchronization problems by using discrete phase rotations and a simple estimation is possible.

The scheme given in [14] modulates the side information over the whole OFDM frame. Hereby, this scheme suffers from an increase in transmit power. The promising results shown in [14] are only valid for very special choices of the parameters. On the contrary, the approach presented in this paper does not lead to an increase of transmit power.

V. Decoding of the Side Information

A. The Transmission Channel for the Side Information

Before the receiver is able to decode the transmitted information, it has to estimate the phase rotation $\boldsymbol{P}_{\text{si}}^{(u)}$ applied on the actual OFDM frame. This estimation has to be accomplished in frequency domain and after equalization of the channel. A block diagram of the receiver-side signal processing is depicted in the right part of Fig. 1.

The received frequency-domain OFDM frame after equalization is denoted by $\boldsymbol{Y} = [Y_d]$ with $Y_d = A_d^{(\check{u})} + N_d$, i.e., the actual chosen signal candidate plus Gaussian noise with $N_d \sim \mathcal{CN}(0, \sigma_n^2/|H_d|^2)$.

In order to extract the information, whether or not one certain subcarrier has been rotated by $\pi/4$, the received signal is taken to the fourth power, only the real part is considered, and it is scaled by $\gamma = \frac{4}{(\sqrt{M}-1)^4}$. Raising the signal to the fourth power has two effects. On the one hand, the $\pi/4$ rotated constellation is converted into a multiplication by minus one ($(\mathrm{e}^{\mathrm{j}\pi/4})^4 = -1$), whereas the original constellation is not affected. On the other hand, together with the scaling γ, the 4-QAM modulated information carrying signal is removed, i.e, $\gamma A_d^4 = -1$ independent from the actual symbol (for $M > 4$ there appears a special case, which is discussed in Sec. V-B.1 in details).

A block diagram of the effective transmission channel between the side information bits $c_d^{(\check{u})}$ and the variable $\tilde{c}_d$ at

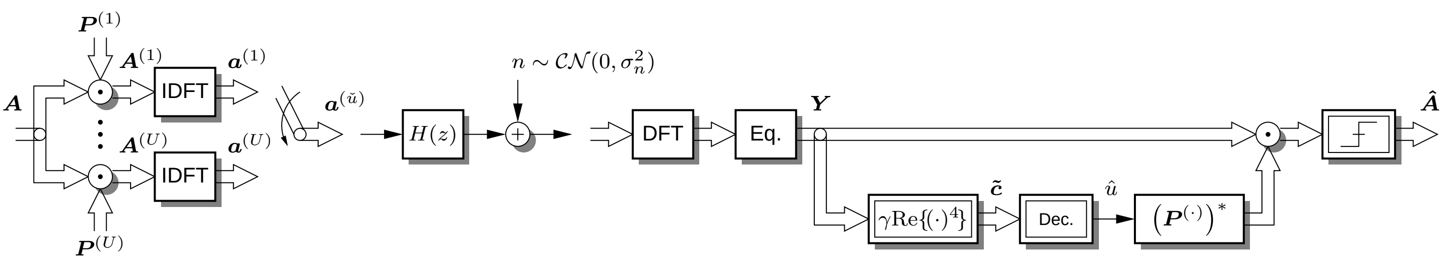

Fig. 1. Block diagram of candidate generation (left), transmission (middle), and decoding and inversion (right) of the side information.

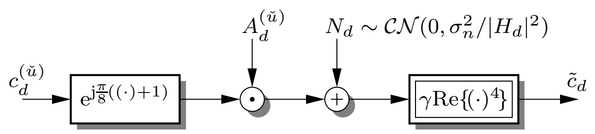

Fig. 2. Block diagram of the effective transmission channel of the side information at each subcarrier.

each subcarrier is given in Fig. 2. According to this figure, the effective channel can be regarded as a special non-Gaussian interference channel.

The $\tilde{c}_d$'s containing the information about the $\pi/4$ phase rotation in the d^{th} subcarrier read

$$\begin{aligned}\tilde{c}_d &= \gamma\mathrm{Re}\{(Y_d)^4\}\\ &= \gamma\mathrm{Re}\{(A_d^{(\check{u})}+N_d)^4\}\\ &= \gamma\mathrm{Re}\{A_d^4\cdot(P_{\mathrm{c},d}^{(\check{u})})^4\cdot(P_{\mathrm{si},d}^{(\check{u})})^4\}+\tilde{N}_d\\ &= \gamma\mathrm{Re}\{A_d^4\cdot 1\cdot c_d^{(\check{u})}\}+\tilde{N}_d\\ &= c_d^{(\check{u})}\cdot\gamma\mathrm{Re}\{A_d^4\}+\tilde{N}_d. \qquad (4)\end{aligned}$$

The effective noise is given by

$$\tilde{N}_d=\gamma\mathrm{Re}\{4(A_d^{(\check{u})})^3N_d+6(A_d^{(\check{u})})^2N_d^2+4A_d^{(\check{u})}N_d^3+N_d^4\}. \quad (5)$$

The probability density function $f_{\tilde{N}_d}(x)$ of the effective noise $\tilde{N}_d$ cannot be calculated easily. Due to the statistical independence of $A_d^{(\check{u})}$ and N_d, it is obvious that $\tilde{N}_d$ has zero mean. Furthermore, the variance can be calculated to

$$\sigma_{\tilde{N}_d}^2=\mathrm{E}\left\{\tilde{N}_d^2\right\}=\gamma^2\left(\frac{72}{35}\cdot\frac{9M^2-40M+51}{(M-1)^2}\cdot\frac{\sigma_a^6\sigma_n^2}{|H_d|^2}+\frac{36}{5}\cdot\frac{7M-13}{M-1}\cdot\frac{\sigma_a^4\sigma_n^4}{|H_d|^4}+48\cdot\frac{\sigma_a^2\sigma_n^6}{|H_d|^6}+12\cdot\frac{\sigma_n^8}{|H_d|^8}\right). \quad (6)$$

B. Preprocessing and Decoding of the Side Information

The task of the receiver is now to find estimates for the actual transmitted side information bits $c_d^{(\check{u})}$ out of $\tilde{c}_d$, $\forall d\in\mathcal{I}_{\mathrm{si},i}$, $i=1\ldots,\mu$. For that, the applied repetition code has to be decoded appropriately.

1) Preprocessing of the Side Information: According to (4) the parameter $\tilde{c}_d$ depends also on the actually transmitted information symbol A_d. Fig. 3 shows the original and rotated constellations for $M=4$- (top left) and $M=16$-QAM (top right) and the resulting points after taking the fourth power and dropping the imaginary part (respective bottom plots).

In case of an 4-QAM constellation the estimation of the side information bits is relatively simple. In this case the fourth power of the signal points, drawn from $\mathcal{A}_4$, results in

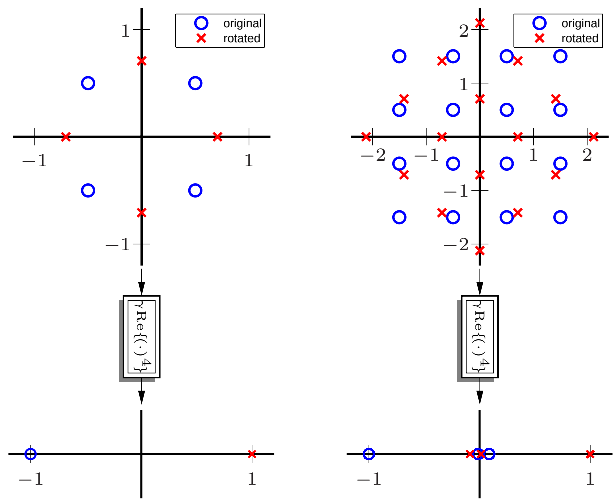

Fig. 3. Top: original QAM constellation (○) and $\pi/4$ rotated QAM constellation (×); bottom: resulting points after taking each point to the fourth power and dropping the imaginary part; $M=4$ (left), $M=16$ (right).

$\gamma A_d^4=-1$. Hence, and according to (4), the decision of the side information bits is not depending on the actual transmitted information carrying signal points, i.e., $\tilde{c}_d=-c_d^{(\check{u})}+\tilde{N}_d$.

In case of higher QAM constellations ($M>4$), the estimation of the side information bits is not as simple because the fourth power of the signal points does not result in a single particular value, i.e., in general $\gamma A_d^4\neq -1$. As it can be seen from the right column of Fig. 3 (for the example $M=16$) there also occur signal points around zero whose meaning is not distinct, as there are points greater than zero which belong to the original constellation and vice versa. Only for the corner points of the constellation the fourth power is -1. Hence, transmission of the side information is only possible if the information carrying signal is one of the four corner points.

In order to be able to estimate the side information bits also in this case, we focus on that situations where the information carrying symbols A_d are corner points. Therefore we introduce an *erasure zone* between $\pm\delta$. If the side information representing parameter $\tilde{c}_d$ falls into this interval, it is marked as an erasure and therefore ignored by the decoder.

Now, an appropriate value δ for the width of the erasure zone has to be determined. According to Fig. 3, the fourth power of the non-corner points is approximately zero. Hence, a first and simple approach would be to set this border half way between the desired signal point and the origin, i.e., $\delta=0.5$.

An optimization of the border δ can be accomplished by regarding the channel capacity of the end-to-end transmission channel of the side information as depicted in Fig. 2 and after ternary quantization of $\tilde{c}_d$ to ± 1 or to erasures. This channel is known in literature as *binary symmetric error and erasure channel (BSEC)* [7] and its channel capacity reads

$$C_{\text{BSEC}} = (1-\varepsilon)\left(1 - \mathrm{e}_2\left(\frac{\beta}{1-\varepsilon}\right)\right) \tag{7}$$

with bit error rate

$$\beta \;=\; \frac{1}{M}\sum_{\forall A_d \in \mathcal{A}_M} \int_{\delta}^{\infty} f_{\tilde{N}_d}(x - \mathrm{Re}\{A_d^4\})\,\mathrm{d}x \tag{8}$$

and erasure rate

$$\varepsilon \;=\; \frac{1}{M}\sum_{\forall A_d \in \mathcal{A}_M} \int_{-\delta}^{\delta} f_{\tilde{N}_d}(x - \mathrm{Re}\{A_d^4\})\,\mathrm{d}x. \tag{9}$$

In order to optimize δ, we have evaluated this capacity for $M = 16$. The evaluation of the integrals in (8) and (9) bases on a numerical analysis of $f_{\tilde{N}_d}(x)$. The top plot of Fig. 4 shows C_{BSEC} for different values of the signal-to-noise ratio $\sigma_a^2|H_d|^2/\sigma_n^2$ and δ. The trajectory of maximum capacity for each value of $\sigma_a^2|H_d|^2/\sigma_n^2$ is depicted in black and also displayed in the bottom plot of Fig. 4 together with the corresponding values δ. Using this curve as a look-up table, the receiver is able to optimize δ in each subcarrier, under the assumption it has full knowledge of the channel state information. The resulting maximum capacity is depicted blue. For high signal-to-noise ratios, the maximum capacity is $C_{\text{BSEC}} = 0.25\,[\text{bit/channel use}]$, because the side information can only be transmitted if the information carrying signal corresponds to one of the four corner points out of the $M = 16$ points of the constellation.

Noteworthy, as this scheme may be used with any QAM constellation, it is also possible to apply bit loading over the carriers. The application of power loading is straightforward.

2) Decoding the Side Information: After getting an estimate of the $\pi/4$ phase rotations within the OFDM frame, the receiver has to decode the index $\check{u}$ of the applied mapping, i.e., find an estimate $\hat{u}$ of it. As mentioned in Sec. IV in this paper we consider a simple repetition code with r repetitions of the binary codeword $\boldsymbol{b}$. Evidently, it is possible to apply a more sophisticated coding scheme. However, the promising numerical results, shown in Sec. VI, justify this approach.

The most simple approach of decoding the index $\check{u}$ given the estimate $\tilde{\boldsymbol{c}}$ is a hard decision at each relevant subcarrier and a majority decision. Possible erasures within $\tilde{c}_d$ are simply not included into the decision.

Moreover, it is also possible to formulate a soft-decision *maximum-likelihood (ML)* estimator for the i^{th} bit of the binary word representing the side information. This estimate reads

$$\hat{b}_i = \underset{b_i=\pm 1}{\operatorname{argmax}} \prod_{\forall d \in \mathcal{I}_{\text{si},i}} f_{\tilde{N}_d}(\tilde{c}_d | b_i, A_d)\,, \quad i = 1,\ldots,\mu. \tag{10}$$

An evaluation of this estimate is not easily possible, as we have no analytical expression of $f_{\tilde{N}_d}(x)$. Moreover, the effective noise $\tilde{N}_d$ is not Gaussian distributed, whereby it is not possible to formulate an additive decision metric according to squared Euclidean distance as common. In order to overcome these issues, we approximate $\tilde{N}_d$ with a real Gaussian random process with zero mean and variance $\sigma^2_{\tilde{N}_d}$ from (6). Evidently, this approximation does not lead to an optimum estimator of

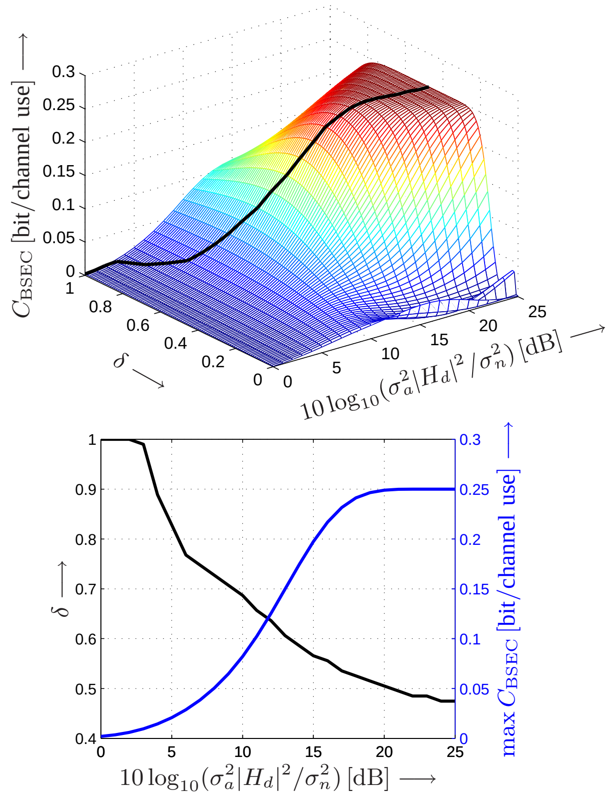

Fig. 4. Top: capacity of BSEC with bit error rate (8) and erasure rate (9) for various signal-to-noise ratios and borders δ of the dead interval; the trajectory of maximum capacity is depicted black; bottom: resulting borders δ (black) and maximum capacity (blue) over signal-to-noise ratio along the trajectory of the top plot; $M = 16$.

the side information bits, but it turns out to be an improvement compared to hard decision. The estimator is hence given by (possible erased values $\tilde{c}_d$ are counted as zero)

$$\begin{aligned}\hat{b}_i &= \underset{b_i=\pm 1}{\operatorname{argmin}} \sum_{\forall d \in \mathcal{I}_{\text{si},i}} \frac{(\tilde{c}_d - b_i)^2}{\sigma^2_{\tilde{N}_d}} \\ &= \underset{b_i=\pm 1}{\operatorname{argmax}}\, b_i \sum_{\forall d \in \mathcal{I}_{\text{si},i}} \frac{\tilde{c}_d}{\sigma^2_{\tilde{N}_d}} \\ &= \begin{cases} +1; & \sum_{\forall d \in \mathcal{I}_{\text{si},i}} \frac{\tilde{c}_d}{\sigma^2_{\tilde{N}_d}} \geq 0 \\ -1; & \sum_{\forall d \in \mathcal{I}_{\text{si},i}} \frac{\tilde{c}_d}{\sigma^2_{\tilde{N}_d}} < 0 \end{cases}.\end{aligned} \tag{11}$$

for $i = 1,\ldots,\mu$.

C. Asymptotic Behaviour of the Error Performance

As mentioned above, for $M > 4$ the side information can only be reliably transmitted if the actual transmitted information carrying symbol A_d is one of the four corner points of the constellation. There may occur situations where for all r repeated positions of one bit b_i no corner point is transmitted. Now, it is not possible to reliably decode the side information, even for high signal-to-noise ratios. In this case the decoder has to guess with probability one half, whether b_i is positive or negative. If this happens for at least one of the μ bits of $\boldsymbol{b}$ than the side information is estimated incorrectly. Hence, for high SNR, the *side information error rate (SIER)* converges to

$$\text{SIER} \rightarrow 1 - \left(1 - \frac{1}{2}\cdot\left(\frac{M-4}{M}\right)^r\right)^\mu. \tag{12}$$

If the side information is estimated incorrectly, than the phase rotation is reverted incorrectly with probability $3/4$ and a bit error ratio of $1/2$ occurs. Hence, for high SNR the *bit error ratio (BER)* converges to

$$\text{BER} \rightarrow \frac{3}{8} \cdot \left(1 - \left(1 - \frac{1}{2} \cdot \left(\frac{M-4}{M}\right)^r\right)^\mu\right). \qquad (13)$$

The lower bounds for the SIER (12) and BER (13) are only depending on the cardinality M of the constellation, the number D of active subcarriers, and the number U of assessed signal candidates (the later two determine the parameters μ and r).

VI. Numerical Results

In Fig. 5 the over-all error performance of this SLM approach is analyzed. On the one hand for $D = 128$ (top part) representing small OFDM frames and on the other hand for $D = 512$ (bottom part) representing large OFDM frames. Thereto, we analyze the bit error ratio (respective top rows) and the side information error ratio (respective bottom rows). For the bit error analysis the reference curve is the bit error ratio of perfect transmission of the side information, i.e., for SLM with $U = 1$ (original signal), where no side information has to be transmitted. As channel model, we consider a $l_{\mathrm{H}} = 5$-tap equal gain Rayleigh fading channel.

Large numbers of subcarriers (here $D = 512$) lead to many possible repetitions r which hence results in a very low SIER. Evidently, the more signal candidates are assessed, the more bits of side information μ have to be transmitted, the less repetitions r are possible, whereby the SIER decreases. However, even for a large number U of assessed signal candidates (here $U = 32$), the BER of SLM converges to the BER of the original signal also for low signal-to-noise ratios.

The performance of hard decision is already that good, that the utilization of soft decision can only lead to moderate gains. Hence, the more simple hard decision is preferable here. For $M = 16$ the flattening of BER or SIER (according to (12) or (13)) is not relevant in the regarded region.

For small numbers of subcarriers (here $D = 128$) and $M = 4$, the results are comparable with the ones discussed above. For $M = 16$ the flattening of the curves can be observed (the horizontal asymptotes are depicted gray). In this case, using soft decision of the side information leads to remarkable gains in BER or SIER performance compared to a hard decision.

In both cases ($D = 128$ and $D = 512$), an optimization of the width δ of the erasure zone leads to neglectable gains in error performance, hence it is sufficient to use the simple approach $\delta = 0.5$.

Fig. 6 shows the asymptotes of SIER or BER ($M = 16$) depending on the number U of assessed signal candidates for different numbers D of subcarriers. For OFDM with large numbers of subcarriers ($D \geq 256$) and practicable numbers of signal candidates ($U \leq 32$), the asymptotic bound of the bit error ratio is less than 10^{-6}.

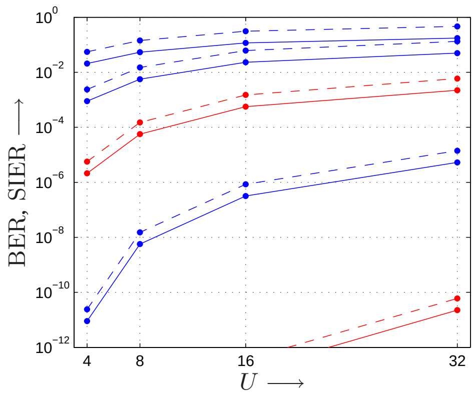

Fig. 6. Asymptotic SIER (dashed) or BER (solid) over the number U of assessed signal candidates for $M = 16$ and (from top to bottom) $D = 32$, 64, 128, 256, 512. The subcarrier sizes analyzed in Fig. 5 are marked in red.

VII. Combination with Extensions of SLM

The approach, described above, for transmitting the side information has been discussed only for single-antenna systems. In the last years the combination of multi-antenna systems with OFDM has become very popular. For these setups special extensions on SLM have been developed in [2], [8], [9]. The novel approach for transmitting the side information can straightforwardly be applied on these extensions as well.

In [10], [11] an alternative scheme for generating different signal candidates, using Reed-Solomon encoding, has been proposed. Moreover, the combination of channel coding with this scheme has been discussed. The embedding of the side information as described above is also possible here, and has to be done after generating signal candidates, encoding, and modulating QAM symbols using the rotation vectors $\boldsymbol{P}_{\mathrm{si}}^{(u)}$.

With *successive SLM* [15] this scheme can also be applied. However, in this case it may be possible to extend the approximated ML estimator (10) to a maximum a-posteriori estimator in order to achieve even better results in error performance.

VIII. Conclusions

In this paper a novel scheme for the transmission of the side information with SLM has been proposed. The side information is embedded into the OFDM frame by combinations of rotated (by $\pi/4$) and not rotated QAM constellations at each subcarrier. Hence, no rate loss and no increase of the transmit power occurs. Moreover, these combinations are chosen that they have no influence on the receiver side synchronization. In order to protect the side information against transmission errors a simple repetition code is used. The numerical simulations show, that the ratio of estimation errors of the side information is very low, which results to a bit error performance almost equal to the case of perfect transmission of the side information.

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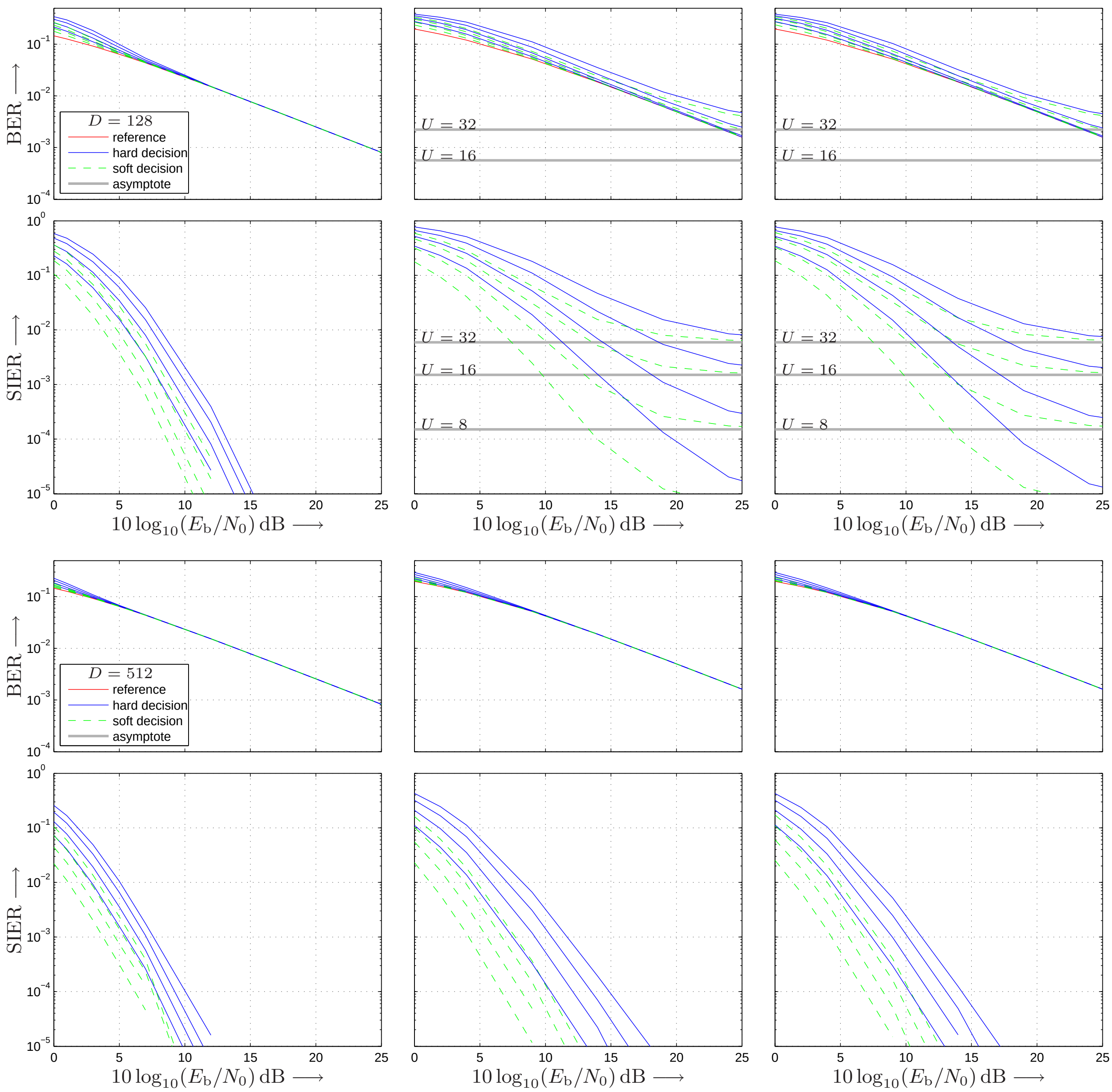

Fig. 5. BER (respective top rows) and SIER (respective bottom rows) over signal-to-noise ratio (E_b/N_0) for SLM assessing $U = 4$, 8, 16, 32 (curves from bottom to top) signal candidates; top plots: $D = 128$; bottom plots: $D = 512$; left column: $M = 4$; middle column: $M = 16$ with $\delta = 0.5$; right column: $M = 16$ with optimized δ according to look-up table from Fig. 4.

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