Noncoherent Continuous-Phase Modulation for DS-CDMA

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Abstract — The combination of continuous phase modulation (CPM) with direct-sequence code-division multiple access (DS-CDMA) for multiuser transmission over the additive white Gaussian noise channel is discussed. Concentrating on the important special case of generalized minimum-shift keying, particularly simple receiver structures are obtained. To emphasize on low complexity, noncoherent detection is proposed and appropriate transmitter and receiver designs are provided. The application of reduced-state noncoherent sequence detection and noncoherent filter adaptation ensures high power efficiency and robustness against channel phase variations. Simulation results confirm that the chosen approach of CPM for DS-CDMA achieves high performance with very moderate complexity.

Keywords — CPM, CDMA, Noncoherent transmission, GMSK

I. INTRODUCTION

Direct-sequence code-division multiple access (DS-CDMA) [1] is an efficient and flexible strategy to accomplish multiuser communication. Considering the uplink in multiuser applications, e.g. in consumer electronics, with particularly high demands on power consumption and price of user devices, continuous phase modulation (CPM) [2] is an attractive technique for combination with DS-CDMA. Due to its constant envelope, low-cost power amplifier are applicable and amplifier nonlinearities are avoided without any back-off. Due to smoothed phase transitions, CPM is highly power and bandwidth efficient, too.

In this paper, we focus on a subset of CPM known as generalized minimum-shift keying (GenMSK) (cf. e.g. [3] and references therein) including both MSK and Gaussian MSK [4]. Since GenMSK can be well approximated as offset quadrature phase-shift keying (OQPSK) [5], [6] simple-to-implement receiver structures result.

We present appropriate transmitter structures for both uncoded and coded CPM transmission, which allow noncoherent reception and preserve the correlation properties of the CDMA spreading sequences. At the receiver, the application of approximate matched filtering and linear minimum-mean square-error (MMSE) multiuser-interference (MUI) and intersymbol-interference (ISI) suppression are devised. To further emphasize on low complexity we propose noncoherent detection avoiding explicit phase-synchronization units. In particular, we apply noncoherent sequence detection (NSD) [7] for improved power efficiency and robustness against phase impairments [8], [9]. Accordingly, MMSE filter adaptation is performed by a noncoherent least-mean-square (NC-LMS) algorithm [10]. In order to fully exploit the statistical properties of the GenMSK signal, the concept of widely linear estimation [11] is adopted.

Simulation results for asynchronous DS-CDMA with MSK and GMSK, respectively, verify that the proposed noncoherent CPM scheme enables power-efficient multiuser transmission with a very modest system complexity.

Finally, we note that the performance of DS-CDMA with coherent GenMSK transmission over the additive white Gaussian noise (AWGN) channel with a conventional correlation receiver has been studied in [12], [13], [14], [15]. In [12], [15] also differential detection is proposed. Whereas in [12] quantization of the channel phase and high-complexity trellis decoding is performed, low power efficient conventional differential detection is applied in [15]. Thus, the presented noncoherent system is clearly superior with respect to complexity and performance.

II. TRANSMISSION SYSTEM

In this section, we introduce the equivalent complex baseband (EBC) system model for noncoherent CPM multiuser transmission. First, a convenient representation of GenMSK signals is provided and the resulting transmitter is given. Then, the assumed channel model and the proposed low-complexity receiver structure are presented.

A. Signal Representation for Binary CPM

In general, binary CPM can be expressed as the superposition of a number of pulse-amplitude modu-
loration (PAM) waveforms, which is known as Laurent decomposition [5]. For the interesting special case of modulation index 1/2, that is GenMSK signaling [3], the differential encoder, which is inherent to the CPM modulator to ensure phase continuity, can be compensated by a binary differentiator in front of the CPM modulator. Such a concatenation of a differentiator and the actual CPM modulator (see Figure 1) is also referred to as CPM with phase-state mapping [16], [9].

Let \( \psi[k] \in \{0, 1\} \) be the differentiator input signal. Then, using only the most important main pulse [5] containing most of the signal power, for \( t \geq L \cdot T \) GenMSK signals are well approximated by [5], [3], [6]

\[
s(t) = \sum_{k=0}^{\infty} i^{(k+1)} \psi[k] \cdot g(t - kT),
\]

where \( T \) is the modulation interval and \( g(t) \) is an impulse of length \((L+1)T\) with a peak at \((L+1)T/2\). By expression (1) CPM is viewed as QOQSK with a special transmit-pulse form. For the most important case of GMSK with time-bandwidth product \( BT = 0.3 \), \( g(t) = c_0(t) \) with \( L = 4 \) is true, where \( c_0(t) \) is defined in [5] (see also [6, Fig. 2]).

\[ B. \ CPM \ for \ DS-CDMA \]

The transmitter block diagram for user \( \mu \), \( 0 \leq \mu \leq K - 1 \), of a K user DS-CDMA system is depicted in Figure 1. The binary data symbols \( a_{\mu}[l] \) are convolutionally encoded yielding the coded symbols \( b_{\mu}[k] \), where the time indices \( l, k \in \mathbb{Z} \) are related via the code rate \( R_c \). Here, we particularly note that a differential encoder with \( R_e \) might be chosen, which can also be regarded as unencoded transmission. Next, the user-specific, real-valued spreading sequence \( s_{\mu[k]} \triangleq [s_{\mu[0]} \ldots s_{\mu[M-1]}]^T \) of length \( M \) is multiplied by \( b_{\mu}[k] \). \( M \) is referred to as the spreading factor. The spread signal \( \psi_{\mu}[k], 0 \leq \mu \leq M - 1 \), is the input for CPM with phase-state mapping yielding the transmit signal \( s_{\mu}(t) \), which can be approximated by (see Eq. (1))

\[
s_{\mu}(t) = \sum_{k=0}^{M-1} \sum_{l=0}^{\infty} j^{i(k-M+i+1)} \psi_{\mu}[k] \cdot g(t - kT_c)
\]

with the chip duration \( T_c = T/M \).

The advantage of CPM with phase-state mapping is that, after compensation of the switch between transmitting over in-phase and quadrature branch at the receiver, apart from the intersymbol interference introduced by the transmit pulse \( g(t) \) the auto- and cross-correlation properties of the spreading sequences are maintained. Of course, by applying the differentialization, phase ambiguities due to the rotational invariance of CPM become unresolvable unless a noncoherently non-catastrophic convolutional encoder [17] is applied [9].

\[ C. \ Channel \]

We consider multiuser transmission over the AWGN channel with a signal phase unknown to the receiver. The continuous-time received signal \( r(t) \) is the superposition of the transmitted signals of \( K \) users:

\[
r(t) = \sum_{\mu=0}^{K-1} \sum_{l=0}^{\infty} e^{j\theta_{\mu}(t)} s_{\mu}(t) + n(t),
\]

where the unknown phases \( \theta_{\mu}(t) \) are slowly time-varying and \( n(t) \) denotes complex-valued AWGN with two-sided power spectral density \( N_0 \) in the ECB domain (corresponding to a physical channel noise with one-sided power spectral density \( N_0 \), as usual).

\[ D. \ Receiver \ Structure \]

Focusing on a simple implementation and low computational complexity, the proposed receiver structure (for user \( \mu \)) is illustrated in Figure 2. Regarding the signal representation (1) a chip matched filter \( g^*(-t) \) is required as receiver input filter \( h(t) \). Since, in general, \( g(t) \) has not a square-root Nyquist characteristic, a noise whitening filter might follow \( g^*(t) \). Both filters can be combined to obtain a whitened matched filter (WMF) [18]. However, a simple square-root raised cosine filter with roll-off factor 0.2 can be employed instead of the optimum WMF in case of GMSK with negligible loss in performance (for details see [19]).

After sampling with chip rate \( T_c^{-1} \) presuming optimal sampling phase for the considered user \( \mu \), the signal is denoted by multiplication with \( j^{-i(k+1)} \), \( k = k \cdot M + i \), (see Eq. (2)). The resulting signal

\[
r_{\mu}(t) = e^{j\theta_{\mu}(t)} s_{\mu}(t) + I_{\text{MU}}[k] + I_{\text{SH}}[k] + n_{\mu}[t],
\]

\[ 0 \leq i \leq M - 1 \), is affected by a phase rotation with \( \theta_{\mu}(k \cdot M + i T) \), multiuser interference (MUI) \( I_{\text{MU}}[k] \) due to the other \( K - 1 \), possibly asynchronous, active
users, intersymbol interference (ISI) $I_{\text{ISI}}[k]$ depending on the shape of $g(t) \ast h(t)$, and AWGN $n[k]$. Of course, since $b_{\mu}[k]a_{\mu}^k$ is real, only the interference and noise components along the line with phase $\theta_{\mu}(k \cdot M + i)T$ are significant.

Next, $r'[k]$ is despread via filtering with $w[k] \triangleq [w_0[k] \ w_1[k] \ldots w_{M-1}[k]]^T$ and sampling with symbol rate $T^{-1}$. Linear filtering with $F \geq M$ is chosen to effectively suppress MUI in an asynchronous CDMA scenario and for equalization of ISI. The symbol-rate signal $q[k]$ is passed to the decoder (see Section IV) and used for adaptation of $w[k]$ as described in the next section.

III. Filter Adaptation

For flexible and robust multiuser communication, the interference-suppression filter $w[k]$ is adapted according to the MMSE criterion applying noncoherent adaptive algorithms as proposed in [10], [20].

To allow for tracking of fast time-varying interference scenarios immediate decisions $b_{\mu}[k]$ of the coded symbols $b_{\mu}[k]$ are required. For this purpose, noncoherent decision-feedback differential detection (DF-DD) (cf. e.g. [21]) is employed, i.e., the decision variable

$$d[k] = q[k] \cdot \left(q_{\text{ref}}[k-1]\right)^*$$

with the reference symbol [20]

$$q_{\text{ref}}[k-1] \triangleq \beta_0 \sum_{\nu=1}^{N-1} \alpha^{\nu-1} q[k-\nu]b_{\mu}[k-\nu]$$

and normalization constant

$$\beta_0 \triangleq \begin{cases} 1/(N-1), & N \geq 2, \\ 1 - \alpha, & N = \infty, \quad 0 \leq \alpha < 1 \\ \frac{1}{1-\alpha}, & 2 \leq N < \infty, \quad 0 \leq \alpha < 1 \end{cases}$$

is built. The reference symbol $q_{\text{ref}}[k-1]$ provides an estimate of the channel phase $\theta_{\mu}(kT)$ applying previous decisions $b_{\mu}[k-\nu]$ and assuming $\theta_{\mu}(t)$ remains constant during $M$ chip intervals. $N, N \geq 2$, and $\alpha, 0 \leq \alpha \leq 1$, are the number of received symbols used for calculation of $d[k]$ and a forgetting factor, respectively. $q_{\text{ref}}[k-1]$ is a generalization of the reference symbols proposed in [22], [23]. Since the useful signal component is expected to be real-valued, the sign of the real part of $d[k]$ gives the estimate $\hat{b}_{\mu}[k]$. Of course, the channel phase can only be estimated up to an ambiguity of $\pi$ using $\hat{b}_{\mu}[k]$. However, a constant phase shift does not affect the performance of noncoherent filter adaptation.

Now, the error signal (Re{·}) denotes the real part of a complex number

$$e[k] = \hat{b}_{\mu}[k] - \text{Re}\{d[k]\}$$

is determined. Using $e[k]$ the filter coefficients are updated via the noncoherent least-mean-square (NC-LMS) [10] (cf. also [20]), i.e., $w[k+1]$ is obtained as

$$w[k+1] = w[k] + \delta_{\text{LMS}} e[k]\left(B_0 q_{\text{ref}}[k-1]\right)^*$$

where $\delta_{\text{LMS}}$ denotes the adaptation step size and

$$F \triangleq \left[r \overline{M-1}[k-1] r^0[k] \ldots r^0[k] r^0[k+1] \ldots\right]^T$$

comprises $F \geq M$ received samples. Here, it is convenient to center the $F$-samples window around $[r^0[k] \ldots r^{M-1}[k]]$ [24]. The results in [24] indicate that the NC-LMS offers a favorable trade-off between performance, i.e., speed of convergence and steady-state error, and computational complexity.

It is worth noting that with the definition of $e[k]$ we apply the concept of widely linear estimation of a real quantity from complex data [11]. Using the alternative definition $e'[k] = \hat{b}_{\mu}[k] - d[k]$ [25], [26], which corresponds to linear estimation, leads to a considerable performance degradation [24].

IV. Noncoherent Sequence Detection

Now, the application of reduced-state NSD is briefly described. For derivation of the NSD metric we assume the unknown channel phase of the considered user $\mu$ to be constant, i.e., $\theta_{\mu}(t) = \theta_{\mu}$. Later on, this restriction is relieved, and in the simulations presented in Section V, the influence of a time-varying phase on the receiver performance is also investigated.

\footnote{For data estimation with NSD phase ambiguities are resolved, of course.}
Presuming that the sum of residual interference and noise can be approximated as an AWGN sequence, the optimum NSD metric for a block of $N_T$ transmitted binary symbols $y_k[k], 0 \leq k \leq N_T - 1,$ over the noncoherent AWGN channel reads [7]

$$\Lambda[N_T] = \text{Re} \left\{ \sum_{k=1}^{N_T-1} q[k] \cdot \hat{b}_k[k] \left( \sum_{m=0}^{k-1} q[m] \hat{b}_k[m] \right)^* \right\}. \quad (11)$$

The $\hat{b}_k[k]$ correspond via encoding to hypothetical trial data symbols $a_k.$ The metric (11) can be written in terms of the incremental metric $\Lambda[k] \triangleq \Lambda[k+1] - \Lambda[k]$ at time $k, 1 \leq k \leq N_T - 1,$ with

$$\Lambda[k] = \text{Re} \left\{ q[k] \cdot \hat{b}[k] \cdot (q_{cr}^e[k-1])^* \right\} \quad (12)$$

and the reference symbol $q_{cr}^e[k-1] \triangleq \sum_{m=0}^{k-1} q[m] \hat{b}_k[m].$

In its present form, $q_{cr}^e[k-1]$ corresponds to an unlimited phase memory, which grows with time $k.$ Hence, a tree search has to be employed for maximization of $\Lambda[N_T].$ Moreover, $\theta_k(t) = \theta_\mu,$ which is usually not true in practice, is indeed required.

Thus, for a feasible implementation of NSD, $q_{cr}^e[k-1]$ in (12) is replaced by $q_{cr}^{N,0}[k-1]$ as defined in (6). In this way, the phase memory is limited through the choice of $N$ and $\alpha,$ respectively. Now, NSD can be performed by the Viterbi algorithm in a trellis representing the memory of the convolutional code and the phase reference. In order to further reduce complexity of NSD (and limit the number of states in case of $N \rightarrow \infty$), we employ per-survivor processing (PSP) [27].

V. SIMULATION RESULTS

For performance assessment of the proposed multi-user transmission with noncoherent coded CPM, simulations of the bit-error rate (BER) have been performed. For coding a 4 state binary rate 1/2 convolutional code (generator polynomials $g_1 = (5), g_2 = (7)$ (base-8 representation)) is employed. We assume a scenario of $K = 5$ equal power users and asynchronous transmission\(^2\). For spreading Gold sequences of length $M = 7$ are adopted. According to the results in [24], an MMSE filter of length $F = 15$ ensures both effective MUI suppression for asynchronous CDMA and sufficient ISI suppression for GMSK with $BT_s = 0.3.$ The filter is adjusted using the NC-LMS algorithm. First, a training sequence is transmitted until the filter is converged. Then, the receiver operates in the decision-directed mode. Viterbi decoding in an 8-state trellis using PSP is considered, i.e., one additional delay element is spent to account for the phase memory. As most common in literature [22], [23], the special cases $\alpha = 1$ and $N \rightarrow \infty,$ respectively, are considered for the reference symbol (6). For the first simulations, the channel phases $\theta_k(t), 0 \leq k \leq N_T - 1,$ are kept constant.

Figs. 3a and b) show BER vs. $10 \log_{10}(E_b/N_0)$ ($E_b$: received signal energy per information bit) for asynchronous multiuser transmission based on noncoherent MSK. As reference curves, BER for the single-user case ($K = 1$) and BER for coherent transmission, respectively, are also plotted. For both forms of the reference symbol the performance of the noncoherent receiver improves with increasing $N$ and $\alpha,$ respectively, and approaches the limit given by idealized coherent reception. The loss compared to the respective single-user bound is about 2 dB and slightly increasing with decreasing $N(\alpha)$.

The same scenario, but now with spectrally more efficient GMSK is considered in Figs. 4a and b). Similar statements as for MSK apply. Comparing the respective curves in the Figs. 3 and 4 we observe that in terms of power efficiency GMSK and MSK are practically indistinguishable. ISI inherent to GMSK is sufficiently suppressed by widely linear MMSE filtering.

Next, the robustness of the proposed receiver against phase variations is discussed. The phase $\theta_k(kT)$ of the desired user at time $k = k \cdot M + i$ is given by

$$\theta_k(kT) = \theta_\mu((k - 1)T) + \Delta(kT). \quad (13)$$

According to commonly used models, the phase difference $\Delta(kT)$ is chosen as (i) constant $2\pi \Delta f T / M$ to describe the effect of a (small) frequency offset $\Delta f$ (normalized to the symbol interval $T$) and (ii) a white Gaussian process with variance $\sigma^2_{\mu}/M,$ i.e., $\theta_\mu(kT)$ constitutes a Wiener process, to account for the influence of phase noise, respectively.

The performance of the proposed system in the presence of frequency offset and phase noise is illustrated in Figs. 5 and 6, respectively. GMSK modulation and asynchronous CDMA with $K = 5$ users are assumed. As can be seen, the sensitivity to phase variations deteriorates with increasing $N(\alpha).$ On the other hand, the achievable power efficiency in the absence of phase variations improves with $N(\alpha).$ Hence, there is a trade-off between power efficiency for constant phase and robustness against phase variations. In practice, the values of $N$ and $\alpha$ have to be carefully adjusted by either taking into account the a priori maximum expected variations of the channel phase or proper adaptation during transmission. It is worth mentioning that the influence on performance due to time-varying channel phases of the interfering users is negligible [24].

\(^2\)In all simulations, a chip-asynchronous system is assumed, where the signal of the interfering user $\mu, 1 \leq \mu \leq 4,$ is delayed by $\mu$ chips with respect to the desired user $\mu = 0.$
VI. CONCLUSIONS

The combination of CPM with DS-CDMA and the proper design of transmitter and receiver structures to accomplish low-complexity multiuser transmission over the AWGN channel are discussed. We concentrate on GenMSK signaling allowing for a simple-to-implement receiver front end. More specifically, widely linear MMSE interference suppression is successfully employed. High power efficiency and robustness against frequency offset and phase noise are achieved by the application of reduced-state NSF and noncoherent MMSE-filter adaptation. The presented simulation results verify the chosen approach of CPM for DS-CDMA. In particular, popular GMSK proves to be a promising alternative to linear modulation schemes for DS-CDMA.

REFERENCES

Fig. 3. BER over $E_b/N_0$ for DS-CDMA with $M = 7$ and MSK. Noncoherent single-user ($K = 1$) and asynchronous multiuser ($K = 5$) transmission. Reference symbols with a) $\alpha = 1$ and b) $N \to \infty$. For comparison: BER for coherent transmission.

Fig. 4. BER over $E_b/N_0$ for DS-CDMA with $M = 7$ and GMSK. Noncoherent single-user ($K = 1$) and asynchronous multiuser ($K = 5$) transmission. Reference symbols with a) $\alpha = 1$ and b) $N \to \infty$. For comparison: BER for coherent transmission.
Fig. 5. BER over normalized frequency offset $\Delta fT$ for DS-CDMA with $M = 7$ and GMSK. Noncoherent asynchronous multiuser transmission: $K = 5$, $10\log_{10}(E_b/N_0) = 8$ dB. Reference symbols with a) $\alpha = 1$ and b) $N \to \infty$.

Fig. 6. BER over phase-noise standard deviation $\sigma_\phi$ for DS-CDMA with $M = 7$ and GMSK. Noncoherent asynchronous multiuser transmission: $K = 5$, $10\log_{10}(E_b/N_0) = 8$ dB. Reference symbols with a) $\alpha = 1$ and b) $N \to \infty$. 