Rate Regions for Multicarrier Point-to-Multipoint Transmission over Channels with Intersymbol Interference

Clemens Stierstorfer, Robert F.H. Fischer

Lehrstuhl für Informationsübertragung, Friedrich–Alexander–Universität Erlangen–Nürnberg, Cauerstraße 7/LIT, 91058 Erlangen, Germany, Email: {clemens,fischer}@LNT.de

Abstract—Point-to-multipoint (downlink) transmission from a central base station to multiple spatially scattered receivers over frequency-selective channels is considered. Multiple-input/multiple-output multicarrier transmission with spatial Tomlinson-Harashima precoding per carrier is compared to other transmission schemes in terms of the achievable rate regions. To derive the rate regions the respective dual uplink schemes are identified and the number of degrees of freedom is analyzed. A virtual uplink model, offering per-tone sorting, is introduced to match the duality. The influence of the degrees of freedom on the achievable rate regions is studied. It can be shown that per-tone sorting does not affect the shape of the rate region, whereas power loading leads to increased regions.

I. INTRODUCTION

We consider communication of a central base station to K spatially scattered mobile terminals; so-called downlink (DL) transmission. Due to the usually small dimensions of the mobile terminals, only the base station is reasonably equipped with multiple antennas (the number of antennas is assumed to be NB ≥ K). A multiple-input/multiple-output (MIMO) channel is established and the received signals at the mobile terminals suffer from multiple access/antenna interference (MAI). Furthermore, as the employed channel is assumed to be frequency-selective, the received signals are additionally corrupted by intersymbol interference (ISI). Accordingly, efficient cancellation of the interference is vital for such transmission schemes. Owing to the scattered nature of the receivers—cooperation amongst them is impossible—joint processing of the signals in order to deal with the interference is only feasible at the central transmitter.

The scenario at hand is a multiuser problem, and can be identified as a so-called vector broadcast channel (BC) [10]. Multiuser channels are in general characterized by a capacity region, the region comprising all achievable rate tuples. The study of these regions has attracted a lot of attention in recent years and remarkable progress has been made [5], [10], [8]. However, most publications are mainly devoted to ultimate information theoretical limits/optimum resource allocation (e.g. [14]) and leave the question of practicable transmission schemes open. In this contribution, we discuss feasible approaches—in particular multicarrier schemes are assessed. For comparison singlecarrier transmission is briefly mentioned.

In Section II, the channel model is given followed by multicarrier transmission in Section III. Section IV briefly reviews the capacity region of vector broadcast channels; Section V introduces the investigated transmission schemes. A discussion of the rate regions and numerical examples are presented in Section VI; Section VII concludes the paper.

II. CHANNEL MODEL

The base station communicates K individual sequences of T-spaced data symbols a[k]|ν, ν ∈ Z, k = 1,...,K, to the K receivers. The data symbols are taken from a signal constellation A with variance σa2 ⊆ E{|a[k]|ν|^2}, ∀k, and combined into a K-dimensional vector 1

\[ a[ν] = [a_1[ν], a_2[ν],..., a_K[ν]]^T \]

From the data symbols the NB-dimensional transmit vector x|ν containing the symbols of the NB ≥ K transmit antennas is generated via some kind of preprocessing. The transmit signals are assumed to meet the sum power constraint E{∥x|ν∥^2} ≤ Kσν^2 = P.

The channels between each pair of transmit and receive antennas are modeled by discrete time impulse responses—given in the equivalent complex base band [6]—which comprise matrix matched filtering, pulse shaping at the transmitter, the continuous time channel, receive filter and T-spaced sampling. We denote the channel transfer functions with H^DL|kl(z), k = 1,...,K, l = 1,...,NB and assume their degree to be limited by L−1. Hereby, the indices denote the lth transmit antenna and the kth receiver, respectively. Employing vector/matrix notation, these transfer functions are combined into a matrix polynomial

\[ H^DL(z) = [H^DL|kl(z)] \]

describing the dispersive MIMO broadcast channel. The K-dimensional vector y|ν of receive signals is obtained as the convolution of the input signal with the channel impulse response plus additive noise n|ν, which is assumed to be Gaussian, and spatially and temporally white with variance σν^2 = N0/T per component.

In the following, we assume block-wise transmission (block length D) and a block-fading channel, i.e., the channel state remains unchanged for at least one block.

III. MULTICARRIER TRANSMISSION

Multicarrier transmission over frequency-selective channels employing orthogonal frequency-division multiplexing

1Notation: A^T: transpose of matrix A; A^H: conjugate transpose; A^{-1}: inverse of the conjugate transpose of square matrix A; I: identity matrix.
(OFDM) is well-known from single-input/single-output (SISO) channels, e.g., [1]. OFDM transforms the dispersive channel into a set of parallel frequency-flat subchannels so-called carriers (we denote the number of carriers with $D$, equivalent to the size of a transmitted block). The OFDM transmitter comprises signal mapping in frequency domain, an inverse discrete Fourier transform (DFT), and the insertion of a guard interval (at least of length $L-1$), whereas the respective receiver consists of guard interval removal and a DFT followed by detection in frequency domain.

This concept can be directly applied to MIMO ISI channels, by equipping each antenna of the MIMO system with an OFDM transmitter and receiver, respectively [3]. Analog to SISO systems, the MIMO ISI channel is decomposed into a set of $D$ parallel carriers, which then are flat-fading MIMO channels. These are related to the transfer function of the MIMO ISI channel by

$$H_d^{DL} = H^{DL}(e^{j2\pi(d-1)/D}),$$

where $d = 1, \ldots, D$ indexes the carrier. The additive noise in each carrier remains white with correlation matrix $\sigma_d^2 I$.

The received signal in carrier $d$ is given by

$$y_d = H_d^{DL} x_d + n_d,$$

where $x_d$ is the vector of transmit symbols (generated from the data symbols in $a_d$) and $n_d$ denotes the additive white Gaussian noise. Canceling the remaining MAI in the $D$ flat-fading MIMO channels with spatially scattered receivers, is a problem which has been repeatedly addressed in literature, e.g., [12], [13].

As the focus of this paper is on maximum achievable rates and error-free transmission, an outer channel code has to be employed. We assume the codewords to span over frequency only and that joint decoding is impossible.

IV. CAPACITY REGION

In order to assess the the aforementioned broadcast scenario, let us first reformulate the receive signal for an entire block as

$$y_{BC} = \sum_{k=1}^{K} H_k x_k + n.$$

Here, $y_{BC}$ and $n$ are $DK \times 1$ vectors containing the stacked receive signals and noise samples of all receivers and carriers, respectively. The user dependent channel matrices $H_k$ are $DK \times D$ dimensional, and the vector $x_k$ contains the transmit signals in all carriers of user $k$.

Regarding this formulation, we can identify the MIMO multicarrier downlink as a vector broadcast channel. For a performance evaluation of this type of channel we have to resort to its capacity region $C_{BC}(P)$, the set of all rate tuples $R$ achievable under the sum power constraint \[\sum_k E\{||x_k||^2\} = \sum_k p_k \leq P.\]

However, due to the non-convex structure of the optimization problem a straightforward derivation of $C_{BC}(P)$ is not possible. Recent literature [5], [10], [8] shows, that the capacity region of (vector) broadcast channels is identical to that of the dual (vector) multiple access channels under the sum power constraint $P$.

The computation of the capacity region of vector MACs subject to individual power constraints (limited power per user, $E\{|x_k|^2\} \leq p_k$, $p = [p_1, \ldots, p_K]$) has been examined e.g., in [9], [15]. In [15] optimum transmit power distributions over frequency aiming at the maximization of the sum rate of the vector MAC are derived via iterative multiuser waterfilling (MUWF). Inner and outer bounds for the vector MAC capacity region $C_{MAC}(p)$ are obtained as well. Relaxing the single user power constraints $p$ to a sum power constraint and taking the convex hull over all resulting vector MAC regions leads to the capacity region of the dual vector broadcast channel

$$C_{BC}(P) = \text{Convex Hull} \{C_{MAC}(p)\}.$$

Here, $P$ denotes the set of all power distributions $p$ fulfilling the sum power constraint $P$.

V. TRANSMISSION SCHEMES AND RATE REGIONS

In the following, we introduce transmission schemes to be evaluated in terms of their achievable rate regions. The main focus is on the application of MIMO precoding in the multicarrier scenario. For comparison, we also present a frequency-division multiple access scheme and a single-carrier precoding approach. Analog to duality for the capacity regions, duality can be exploited for the derivation of the rate regions of the investigated schemes. I.e., the rate regions are not directly derived from the presented downlink transmission, but dual uplink (UL) schemes are identified and employed for further computations.

A. Multicarrier Precoding

1) Concept: First, consider the multicarrier channel, i.e., the set of $D$ independent and frequency-flat MIMO channels. By means of individual precoding in each carrier MAI in the received signals can be prevented. Here, the focus of our investigations is on spatial Tomlinson-Harashima precoding (THP) [13], cf. Fig. 1. However, for sake of brevity, we do not go into much detail, but immediately proceed to the respective dual uplink transmission.

2) Dual Uplink Concept: By flipping the entire precoding scheme we obtain a structure which can be identified as MIMO decision-feedback equalization (DFE, aka. V-BLAST). The $d$th channel matrix $H_d$ is the Hermitian of the downlink channel matrix $H_d^{DL}$ (for a detailed description of the relations among uplink and downlink matrices see [3]). The usage of an arbitrary power distribution $p$ fulfilling the sum power constraint $P$ can be taken into account by appropriate scaling of the columns of the channel matrices leading to $H_d$.

2 Actually, these proofs only refer to the dirty paper coding regions. In [11] the equivalence of dirty paper coding region and the capacity region of BCs has been shown.

3 Selected quantities taking a particular power distribution $p$ into account carry a tilde ($\tilde{\cdot}$).
The filter matrices computed according to the minimum mean-squared error (MMSE) criterion are given by [3]

\[ P_d^T \left( \tilde{H}_d^H \tilde{H}_d + \zeta I \right) P_d = B_d^H \Sigma_d B_d, \quad (5) \]

with \( \zeta = \sigma_n^2 / \sigma_a^2 \), the diagonal matrix \( \Sigma_d = \text{diag}(\tilde{s}_d, \ldots, \tilde{s}_d, K) \), and the lower triangular feedback matrix \( B_d \) with unit main diagonal.\(^4\) The feedforward matrix calculates to [3]

\[ F_d = \Sigma_d^{-1} B_d^{-1} P_d^T \tilde{H}_d^H. \quad (6) \]

Ignoring the error propagation in the feedback loop, the MIMO ISI channel is decomposed into \( DK \) independent subchannels with signal-to-noise ratios

\[ \text{SNR}_{d,k}^{(P_d)} = \frac{\sigma_a^2}{\sigma_n^2 \Sigma_{d,k}} - 1. \quad (7) \]

3) Virtual Uplink: Comparing downlink and dual uplink scheme reveals additional degrees of freedom in the downlink. In the uplink the successive cancellation in the spatial dimension relies on immediate decisions of transmitted symbols. Since these decisions can only be obtained from decoded codewords, all symbols of a particular user have to be processed at the same cancellation step. Consequently, an individual sorting of the users in the carriers is prevented \((P_d = P, \forall d)\). In the downlink no decisions are needed for the encoding process, as the transmitter has perfect knowledge of all symbols. Hence, causality in the successive processing is not violated by a per-carrier sorting of the users. Evidently, the downlink offers additional degrees of freedom, which could be exploited for some kind of optimization.

In order to cope with this inconsistency of uplink and downlink model, we introduce a virtual uplink model. This model also employs spatial DFE in each carrier, but individual sorting is feasible—the causality constraint is simply neglected. In terms of usable degrees of freedom, this virtual uplink model fully matches its downlink counterpart.\(^5\)

\(^4\) Obviously, the resulting filters and matrices depend on the particular choice of the permutation matrix \( P_d \). A dependency on a particular set of orderings in the carriers is, if necessary, emphasized by \( \Pi = \{ P_d \} \).

\(^5\) We are conscious of a discrepancy in the presented concept of duality, since it does not exactly match for THP and DFE, in particular in the MIMO case. The non-Gaussianity of the practical signaling and the modulo character of the channel for THP could be either taken into account by a function mapping SNRs onto rates [2] or by some kind of penalty term [4]. The region obtained assuming perfect duality can be seen as an outer bound for THP.

4) Rate Regions and Power Loading: Based on the SNRs derived in (7) and assuming Gaussian signaling, individual rates can be computed for each user

\[ \tilde{R}_k^\Pi = \frac{1}{D} \sum_{d=1}^D \log_2(1 + \text{SNR}_{d,k}^{(P_d)}). \quad (8) \]

The respective downlink rate region is then obtained by considering all admissible power distributions \( p \in P \) fulfilling the sum power constraint

\[ R^\text{MC}(P) = \text{Convex Hull} \{ R^\Pi_k, \ldots, R^\Pi_{K,d} \}. \quad (9) \]

Multicarrier transmission easily allows the application of power loading over frequency which is compulsory for maximizing the rate region. This might, e.g., be singleuser waterfilling (SUWF) over frequency, multuser waterfilling, or constant power spectral densities (PSD). The sum power constraint \( P \) and the individual power constraints \( p \) do not impose any restrictions on the shape of the transmit PSDs. However, obeying particular power distribution policies over frequency of course affects the shape of the rate regions.

B. Alternative Transmission Schemes

1) Orthogonal Frequency-Division Multiple Access: For comparison, we consider the application of a frequency-division multiple access (FDMA) scheme, where each user is exclusively assigned a certain frequency band and the entire bandwidth is used. This leads to \( K \) independent singleuser transmissions with multiple transmit antennas. The respective rate region is obtained by considering all possible frequency assignments (for a detailed description, we refer to [3]). In this scenario, power loading strategies over frequency can be easily employed.

2) Singlecarrier Precoding: MAI and ISI can be jointly dealt with by spatio-temporal Tomlinson-Harashima precoding. Again the respective dual uplink transmission—spatio-temporal DFE—is exploited to determine the rate region. Its inferiority to multicarrier transmission except in terms of the sum rate has been shown, cf. [3]. The
application of power loading is not straightforward in singlecarrier schemes.

VI. DISCUSSION AND NUMERICAL EXAMPLES

In this Section we discuss the above introduced rate regions and present exemplary numerical results to illustrate the relations. Hereby, we restrict ourselves to $K = N_B = 2$ users and base station antennas (2 × 2 MIMO channels). We assume a four tap equal gain channel which is decomposed into $D = 64$ carriers.\(^6\)

A. Per-Carrier Sorting

The multicarrier downlink provides a larger number of degrees of freedom as does the uplink, since per-carrier sorting is feasible. One might suspect that these additional degrees of freedom could be utilized for enlarging the rate region. Interestingly, employing per-carrier sorting does not affect the shape of the rate region, but yields exactly the same rate region as if merely global sorting was applicable.

This can be easily seen by comparing the conventional and the virtual uplink for an arbitrary, but fixed power distribution $p$. The rate region of the conventional uplink is determined by the $K!$ global sortings, all leading to the same maximum sum rate. Each sorting yields a vertex of the so-called dominant face of the polymatroid representing the respective rate region. Global sorting can be shown to lead to extremal individual rates\(^7\),\(^3\); rate tuples achieving the maximum sum rate without any extremal individual values can be obtained from time-sharing. As the sum rate is independent of a particular sorting, also per-carrier sorting leads to tuples with maximum sum rate, lying on the dominant face and not enlarging the rate region. Hence, per-carrier sorting can avoid the necessity of time-sharing when aiming at particular rate tuples, but

$$R_{UL}^{MC}(p) = R_{UL}^{MC}(p),$$

(12)
does hold for any $p \in P$. Since $R_{UL}^{MC}(P)$ is bounded by the convex hull over all $R_{UL}^{MC}(p)$, this directly translates to the rate region of the MC downlink.

An interesting conclusion from these considerations is, that points on the boundary of any MIMO multicarrier downlink rate region cannot be achieved by global sorting only. Individual sorting is in general necessary for almost all points with maximum sum rate, i.e., the dominant face except its vertices (the latter $K!$ points can be achieved by global sorting).

The case is illustrated for a two user scenario in Fig. 2. Here, on the left-hand side, the rate regions of the uplink (conventional and virtual) are depicted for an equal power distribution and flat transmit PSDs (the describe phenomena hold for arbitrary $p$ and PSDs). Clearly, the boundary of the rate region (blue line) is determined by the two points obtained from global sorting (black circles); they represent the vertices of the dominant face (here: 1-dimensional hyperplane). For sake of clarity, only a selection of rate tuples resulting from per-carrier sorting is represented (red crosses). Showing all $2^6$ rate tuples fails due to complexity, but these tuples would cover almost the entire dominant face. Hence, the time-sharing argument for achieving points on the dominant face of the rate region can be dropped in the multicarrier downlink.

On the right-hand side of Fig. 2 the respective downlink rate region (bounded by blue line) and some exemplary underlying uplink rate regions (gray) are depicted. Here, almost all points (rate tuples) on the boundary of the downlink region can be achieved by global sorting, except those lying on the dominant face (red line). The latter result from per-carrier sorting and the application of the power distribution $p$ leading to maximum sum rate. The rate tuples achieving the maximum sum rate and at the same time showing extremal individual rates—the vertices of the dominant face—again result from global sorting (black circles).

B. Power Loading

The users’ power distributions can be optimized over frequency. Since in the downlink only the sum power constraint $P$ has to be met by the transmit PSDs, various power distribution policies can be applied. These might either aim at maximum sum rate, maximum individual rates or at any particular point in the rate region. However, such strategies may vary significantly in complexity, e.g., employing iterative MUWF in order to obtain PSDs leading to the maximum sum rate is rather costly. A computationally cheaper approach—not falling too much behind the aforementioned one—is the application of singleuser waterfilling according to the SNRs obtained from (7). As the gains due to power loading vanish for higher SNRs, only low to moderate SNR scenarios are of interest in this context. Furthermore, power loading requires channel state information at the receivers.

\(^{6}\)The presented results entirely reflect the phenomena and effects which can be observed for larger numbers of carriers. The channel matrices, drawn from an equal-gain distribution, are given as

$$H_0 = \begin{bmatrix} 0.36 & 0.19 & -0.07 & 0.07 \\ 0.12 & 0.23 & -0.03 & 0.61 \end{bmatrix}, H_1 = \begin{bmatrix} 0.55 & 0.07 & 0.60 & -0.19 \\ -0.11 & 0.34 & -0.30 & 0.29 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.08 & 0.30 & -0.02 & 0.41 \\ -0.06 & 0.19 & -0.65 & 0.31 \end{bmatrix}, H_3 = \begin{bmatrix} 0.48 & 0.29 & -0.48 & 0.18 \\ -0.18 & 1.14 & 0.40 & 0.58 \end{bmatrix}. (11)$$
In Fig. 3 the downlink rate regions are depicted for an exemplary two user case at an SNR of $10\log_{10}(\sigma_n^2/\sigma_1^2) = 1$ dB. Several schemes are compared assuming flat transmit PSDs and power loading approaches. Regarding flat PSDs it can be seen, that FDMA (dashed green) can neither compete with multicarrier transmission (dashed blue) nor with singlecarrier transmission (dashed magenta) in terms of a maximized sum rate. In contrast the boundaries of the multicarrier rate region and of the singlecarrier rate region in parts coincide at points of maximum sum rate, cf. left hand side of Fig. 4. The right hand side of Fig. 4 points out, that all three unloaded schemes achieve identical maximum individual rates. Due to the flat transmit PSDs none of the three approaches approaches the boundary of the capacity region (solid red) obtained by iterative MUWF [15].

The rate regions clearly can be enlarged by applying power loading, cf. Fig. 3. The FDMA rate region (solid green) gains from waterfilling over frequency, but due to the not utilized chance of spatial multiplexing, the gap to other schemes in the maximum sum rate remains. MC transmission employing SUWF after the calculation of the filter matrices assuming an equal power distribution over users and frequency (solid blue) is not far from the capacity region (solid red). Whereas MC transmission with MUWF and taking the optimized PDS at the filter computation into account (dash-dotted black) can achieve the maximum sum rate, cf. left hand side of Fig. 4. However, the boundary of the respective rate region does not entirely coincide with the capacity region of the channel. Maximum individual rates can be achieved by all of the three schemes, when the entire power and bandwidth (in case of FDMA) are exclusively assigned to a user and power loading over frequency is employed, cf. right hand side of Fig. 4.

VII. CONCLUSION

MIMO multicarrier transmission offers a tremendous amount of degrees of freedom to optimize or adapt transmission. This can be clearly seen as the major advantage over other transmission strategies as FDMA or singlecarrier transmission. Although, not all of these degrees of freedom directly translate into increased rates and hence enlarged rate regions. Per-carrier sorting has been shown to lead to a greater flexibility in achieving points with maximum sum rate and hence supersedes the need for time-sharing, but cannot expand the rate region. The latter however can be achieved by applying power loading over frequency.

REFERENCES