Reducing the Peak–to–Average Power Ratio of Multicarrier Modulation by Selected Mapping

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Abstract — A new method for the reduction of peak–to–average transmit power ratio of multicarrier modulation systems, called selected mapping, is presented, which is appropriate for a wide range of applications. Significant gains can be achieved by selected mapping whereas complexity remains quite moderate.

1 Introduction

Multicarrier modulation, often also denoted as orthogonal frequency-division multiplexing (OFDM), has been proposed for many applications. In OFDM transmission schemes a block of $D$ distinct complex-valued carriers $V[k]$, $\mu = 1 : D$,\footnote{R. Bäuml is now with: Ericsson Eurolab, Nürnberg.} (called OFDM frame) is transformed into time-domain using the inverse discrete Fourier transform (IDFT). The main advantage of OFDM is that, by introducing a “guard-period” \cite{1} and using differential encoding, reliable transmission over spectrally shaped channels is possible without any equalization. Unfortunately, because of the statistical independence of the carriers the corresponding time-domain samples $v[k]$, $k = 1 : D$, in the equivalent complex-valued low-pass domain \cite{2} are approximately Gaussian distributed. This results in a high peak–to–average power ratio (PAR)

$$\text{PAR} = \frac{\max_k \{ |v[k]|^2 \}}{\mathbb{E}\{ |v[k]|^2 \}}.$$  \hspace{1cm} (1)

\footnote{$i = 1 : I$ is written instead of $i = 1, 2, \ldots, I$.}
Hence, to avoid non-linear distortion and spectral spreading of the transmit signal highly linear amplifiers operating with a large back-off have to be used.

In many papers methods to reduce the PAR are proposed. But either complexity and redundancy are high and/or only small gains in PAR are achieved. In this paper, a new simple method is introduced, which leads to considerable gains not only asymptotically, but in PAR regions relevant in practice.

2 Selected Mapping

Let \( v[k], k = 1 : D \), be identically and independently two-dimensionally Gaussian distributed. Then, the probability that at least for one sample \( |v[k]|^2 > Y \cdot \mathcal{E}\{|v[k]|^2\} \) is valid, or equivalently, that the PAR of the OFDM frame exceeds a certain threshold \( Y \) reads:³

\[
\Pr\{\text{PAR} > Y\} = 1 - (1 - e^{-Y})^D, \quad Y > 0.
\]  

(2)

Now, assume that \( N \) statistically independent OFDM frames represent the same information. Selecting the frame with the lowest PAR for transmission, the probability that \( \text{PAR}_{\text{low}} \) exceeds \( Y \) is given by

\[
\Pr\{\text{PAR}_{\text{low}} > Y\} = (\Pr\{\text{PAR} > Y\})^N.
\]  

(3)

Figure 1 shows the probability \( \Pr\{\text{PAR}_{\text{low}} > Y\} \) for \( N = 2^l, l = 0:7, \) and \( D = 128 \). Obviously, PAR is concentrated at lower values \( Y \) for increasing \( N \). For clipping at most one out of \( 10^6 \) frames, which can be neglected in practice, even for \( N = 4 \) a gain higher than 3 dB results.

Because of the varying assignment of data to the transmit signal, we call this principle selected mapping. The core is to choose one particular signal which exhibits some desired properties out of \( N \) signals representing the same information.

Now, the question is how to generate OFDM frames representing the same information. Here, we present one possible, very promising solution. Define \( N \) distinct vectors \( \mathbf{P}^{(n)} = [P_1^{(n)}, \ldots, P_D^{(n)}] \), with \( P_\mu^{(n)} = e^{j\varphi^{(n)}}, \ \varphi^{(n)} \in [0, 2\pi), \mu = 1 : D, n = 1 : N \). After mapping the information to the carriers \( V[\mu] \), each OFDM frame is multiplied carrierwise with the \( N \) vectors \( \mathbf{P}^{(n)} \), resulting in a set of \( N \) different frames with components

\[
V[n][\mu] = V[\mu] \cdot e^{j\varphi^{(n)}}, \quad \mu = 1 : D, \quad n = 1 : N.
\]  

(4)

³Please notice, that \( \Pr\{\text{PAR} > Y\} \) is the complementary cumulative distribution function of PAR.
Then, all $N$ frames are transformed into time-domain and the one with the lowest PAR is selected for transmission.

Simulation results for $D = 128$ and 4-ary PSK in each carrier are plotted in Fig. 2. The PAR of such a scheme is reduced according to the prediction from theory. Here, $N = 4$ vectors $\mathbf{P}^{(n)}$ are generated randomly with $P_{\mu}^{(n)} \in \{\pm 1, \pm j\}$. The advantage of using only phase shifts by multiples of $\pi/2$ is that they can be implemented without any multiplications.

Additionally, differential encoding may be applied, taking place before the IDFT and right after generating the equivalent OFDM frames. Then, at the receiver, after performing the DFT differential demodulation has to be implemented.

3 Receiver

In order to recover data, the receiver has to have knowledge which vector $\mathbf{P}^{(n)}$ has actually been used. The straightforward method is to transmit the number $n$ of the vector as side information to the receiver. As this number is of highest importance, side information should be protected by channel coding. Even for simple block codes and moderate codeword lengths erroneous detection of $\mathbf{P}^{(n)}$ can be neglected.

For channel encoded data no side information is necessary at all. Here, $N$ channel decoders process the received signal in parallel. Each of them assumes one specific vector $\mathbf{P}^{(n)}$. Finally, the most probable decoding result is selected to recover data.

4 Conclusions

The proposed scheme for the reduction of PAR can be used for arbitrary numbers of carriers and any signal constellation. Selected mapping provides significant gains at moderate additional complexity. Actually, it is appropriate for all kinds of multiplex techniques, which transform data symbols to the transmit signal. Even in single-carrier systems where PAR grows as the roll-off factor of the pulse shaping filter decreases, selected mapping can be applied advantageously.
References


Captions

Figure 1: Complementary cumulative distribution function of PAR, if the frame with the lowest PAR is selected out of $N$ statistically independent frames.

Figure 2: Complementary cumulative distribution function of $\text{PAR}_{\text{low}}$. $N = 1$ and 4, $D = 128$ and quaternary phase shift keying in each carrier. Solid lines: simulation results, dashed lines: approximations according to Eq. (3).