

Selected Mapping with Explicit Transmission of Side Information

Christian Siegl and Robert F.H. Fischer

Lehrstuhl für Informationsübertragung, Friedrich–Alexander–Universität Erlangen–Nürnberg

Cauerstrasse 7/LIT, 91058 Erlangen, Germany, Email: {siegl, fischer}@LNT.de

Abstract—The high peak-to-average power ratio (PAR) of the transmit signal is one serious issue, which occurs in orthogonal frequency-division multiplexing (OFDM). Hence, the application of a PAR reduction algorithm, which controls the peak-power of the transmit signal, is indispensable. In this paper, we consider selected mapping (SLM), a very popular technique for PAR reduction and its recently proposed extension successive SLM. One drawback of (successive) SLM is that the transmission of side information is necessary, which is extraordinarily prone to transmission errors. In this paper, the explicit transmission of this side information is considered. In order to reduce the increase in bit error rates, due to an erroneous detection of the side information, its estimation at the receiver and its mapping to signals embedded into the OFDM frame is optimized. In this case the application of successive SLM is very advantageous compared to the original approach of SLM as the detection error rate can be reduced significantly. Moreover, with successive SLM almost no differences between explicit transmission and perfect knowledge of the side information in terms of bit error rates occur.

I. INTRODUCTION

Modern communication systems demand for high data rates and hence large signal bandwidths. Digital transmission over channels with large bandwidths causes intersymbol interference, which has to be equalized at the receiver. Due to its simple implementation, over the last years *orthogonal frequency-division multiplexing (OFDM)* [5] has become a very popular scheme for the transmission over such channels.

One of the most serious problems of OFDM is the large *peak-to-average power ratio (PAR)* of the transmit signal. The non-linear power amplification of such a signal at the transmitter front-end leads to signal clipping and hence signal distortion. An even more serious issue, caused by signal clipping, is out-of-band radiation. To avoid the violation of spectral masks the power amplifier has to operate at large power back-offs, which reduces the power efficiency. In order to increase the power efficiency, an algorithmic control of the peak-power of the signal at the transmitter is required. Over the last years a variety of so-called PAR reduction algorithms has been developed. Only to mention the basic existing ideas there are schemes based on multiple signal representations as *selected mapping (SLM)* [3], [15] or *partial transmit sequences (PTS)* [14], [15], *tone reservation* or *tone injection* [22], [12], *active constellation extension* [13], or schemes based on coding techniques [20], [23], [19]. All these PAR reduction schemes have in common, that they introduce some computational complexity at the transmitter

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and influence the signal by increasing the transmit power and/or introduce some redundancy.

In this paper, schemes based on multiple signal representations are considered. Recently many extensions of such schemes have been discussed, e.g., [17], [8]. Subsequently, the extension of the original approach of SLM [3] to *successive SLM (SSLM)* [21] is studied in detail.

All variants of SLM have in common that they require the transmission of extra information which is especially prone to transmission errors. One possible strategy is to disperse this extra information over the signal in order to avoid its explicit transmission [6], [16], [11]. Other approaches try to avoid the transmission of this extra information at all [10], [4], [1].

In this paper, we consider the explicit transmission of side information and show that this is feasible with negligible increase in error rate of the information carrying signal. Our approach depends on a suited receiver-sided estimation of this extra information. Hereby, we consider SLM, but our studies can straightforwardly be applied to PTS, too.

This paper is organized as follows: Section II gives an introduction to the system model. In Section III a short review of SLM and its recently proposed extension SSLM is given. Section IV considers the insertion of the side information into the transmit signal with an optimized mapping and the estimation of the side information at the receiver. Section V shows numerical results and conclusions are drawn in Section VI.

II. SYSTEM MODEL

Subsequently, we consider single-antenna OFDM transmission. The frequency-domain OFDM frame is given by the vector¹ $\mathbf{A} = [A_d]$ with $d = 0, \dots, D-1$. Each signal element A_d is drawn from an M -ary (square) QAM constellation \mathcal{A} with $A_d \in \{\pm A_I \pm jA_Q\}$ with $A_{I/Q} \in \{\frac{1}{2}, \frac{3}{2}, \dots, \frac{\sqrt{M}-1}{2}\}$.

This frequency domain OFDM frame is then transformed into time domain by the *inverse discrete Fourier transform (IDFT)*. The resulting time-domain OFDM frame is written as the vector $\mathbf{a} = [a_k]$ with $k = 0, \dots, D-1$, whereby the elements of \mathbf{a} are determined by $a_k = 1/\sqrt{D} \sum_{d=0}^{D-1} A_d e^{2\pi jkd/D}$. The variance of the time-domain signal is given by $\sigma_a^2 \stackrel{\text{def}}{=} E\{|a_k|^2\}$.

¹Notation: Throughout this paper vectors are denoted with bold letters. Frequency-domain symbols are written as capital letters, time-domain symbols as lower-case letters. We distinguish between the natural logarithm $\log(x) \stackrel{\text{def}}{=} \log_e(x)$, whereby e denotes Euler's number, and the logarithm to base 10: $\log_{10}(x)$. $|\cdot|$ of a (complex) number denotes its absolute value; $\|\cdot\|$ of a vector denotes its Euclidean norm and $\lceil \cdot \rceil$ specifies rounding to the next integer towards infinity.

The time-domain signal² is then transmitted over an AWGN multi-path channel. The variance of the additive (complex-valued) noise is given by σ_n^2 . At the receiver the (complex-valued) fading coefficients H_d in all subcarriers are compensated by means of equalization.

Due to the superpositioning of the carriers, the transmit symbols a_k exhibit a high peak-to-average power ratio (PAR), defined as³

$$\text{PAR} \stackrel{\text{def}}{=} \frac{\max_{k=0, \dots, D-1} |a_k|^2}{\sigma_a^2}. \quad (1)$$

As performance measure, we subsequently consider the *complementary cumulative distribution function (ccdf)*, i.e., the probability that the PAR of a given OFDM frame exceeds a certain threshold PAR_{th} . Under the assumption that the transmit signal is Gaussian distributed, the ccdf of the original OFDM signal reads [3]

$$\begin{aligned} \text{ccdf}_{\text{orig}}(\text{PAR}_{\text{th}}) &= \Pr\{\text{PAR} > \text{PAR}_{\text{th}}\} \\ &\stackrel{\text{Gauss}}{=} 1 - (1 - e^{-\text{PAR}_{\text{th}}})^D. \end{aligned} \quad (2)$$

III. SELECTED MAPPING FOR PAR REDUCTION

A. Original Approach

In [3], the original approach of SLM has been proposed. Given the original OFDM frame, via \hat{U} different bijective mappings $\mathcal{M}^{(u)}$, $u = 1, \dots, \hat{U}$, \hat{U} different signal candidates are generated. Hereby, it is essential, that the mapping does not influence the signal grid, i.e., the mapped signal has to be drawn from the same modulation alphabet. Otherwise the receiver will not be able to synchronize and equalize the received signal in the same way as without PAR reduction. Thereto, in [3] the mapping is performed by an element-wise multiplication with randomly chosen phase vectors $\mathbf{p}^{(u)} = [p_d^{(u)}]$, $d = 0, \dots, D-1$, $u = 1, \dots, \hat{U}$, with $p_d^{(u)} \in \{\pm 1, \pm j\}$. In the following, we always apply this mapping. However, other possibilities (e.g., permutations [9]) are imaginable, too.

Each of the resulting signal candidates is then transformed into time domain and its PAR value is determined. Then, the “best” signal representation, i.e., the one exhibiting the lowest PAR, index u^* , is chosen for transmission. A block diagram of this approach is depicted in Fig. 1.

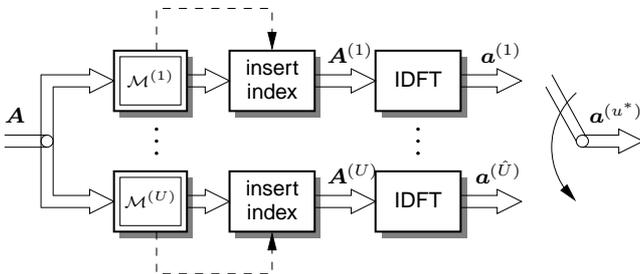


Fig. 1. Block diagram of selected mapping.

²For convenience, we do not consider appending the cyclic prefix, pulse shaping, and modulation to radio frequency.

³Noteworthy, we consider the PAR of the non-oversampled signal, which is sufficient as we are only interested in the relation of the performance between different schemes [21].

Assuming that the signal candidates are pairwise statistically independent, the ccdf of PAR of SLM only depends on the PAR distribution of the original signal (2) and is given by [3]

$$\text{ccdf}_{\text{SLM}}(\text{PAR}_{\text{th}}) = (\text{ccdf}_{\text{orig}}(\text{PAR}_{\text{th}}))^{\hat{U}}. \quad (4)$$

In order to perform the inverse mapping $(\mathcal{M}^{(u)})^{-1}$ at the receiver the transmission of side information—here the index u^* —is required. For that, at least $\lceil \log_2(\hat{U}) \rceil$ bits have to be spend. Hereby, the side information has to be protected extraordinarily as errors of the index u^* will lead to a loss of the entire OFDM frame.

B. Successive Approach

An extension to SLM has been presented in [21], where a threshold PAR_{tol} of tolerated PAR is defined. If the SLM algorithm finds an OFDM frame whose PAR falls below this threshold after only assessing $u < \hat{U}$ signal candidates it stops and no more signal representations are considered. If after \hat{U} trials no alternative signal representation with $\text{PAR} < \text{PAR}_{\text{tol}}$ is found, the candidate exhibiting the lowest PAR value is chosen for transmission.

Given the probability $C_{\text{tol}} \stackrel{\text{def}}{=} \text{ccdf}_{\text{orig}}(\text{PAR}_{\text{tol}})$, in [21] it is shown, that the probability $\Pr\{u\}$ that the u^{th} mapping $\mathcal{M}^{(u)}$ is applied to obtain the best signal candidate is given by

$$\Pr\{u\} = C_{\text{tol}}^{u-1}(1 - C_{\text{tol}}) + C_{\text{tol}}^{\hat{U}}/\hat{U}, \quad u = 1, \dots, \hat{U}. \quad (5)$$

Compared to the original approach of SLM, with this scheme the average number of assessed candidates per OFDM frame and consequently the required computational complexity decreases significantly. The average number of considered candidates per OFDM frame reads

$$E\{U\} = \frac{1 - C_{\text{tol}}^{\hat{U}}}{1 - C_{\text{tol}}} \left[\frac{\text{assessed cand.}}{\text{OFDM frame}} \right]. \quad (6)$$

Interestingly, the average number of assessed signal candidates at the *critical tolerated threshold* $\text{PAR}_{\text{tol}} = \log(D)$ can be very well approximated by Euler’s number $e = 2.781 \dots$ [21]. Given the ccdf of the PAR values of the original signal the ccdf of SSLM reads [21]

$$\text{ccdf}_{\text{SSLM}}(\text{PAR}_{\text{th}}) = \quad (7)$$

$$\begin{cases} \frac{\text{ccdf}_{\text{orig}}(\text{PAR}_{\text{th}}) - C_{\text{tol}}}{1 - C_{\text{tol}}} (1 - C_{\text{tol}}^{\hat{U}}) + C_{\text{tol}}^{\hat{U}}, & \text{PAR}_{\text{th}} < \text{PAR}_{\text{tol}} \\ (\text{ccdf}_{\text{orig}}(\text{PAR}_{\text{th}}))^{\hat{U}}, & \text{PAR}_{\text{th}} \geq \text{PAR}_{\text{tol}} \end{cases}$$

IV. EXPLICIT TRANSMISSION OF THE SIDE INFORMATION

A. Discussion

In order to decode the signal at the receiver, the transmission of side information (the index of the actually applied mapping) is indispensable. The side information is extraordinarily prone to transmission errors as the information of the whole OFDM frame will be lost if the wrong inverse mapping is applied.

In literature there exist various techniques to transmit the side information. In [6], the scrambler variant of SLM has been proposed. The side information is not transmitted explicitly, but is dispersed over the whole OFDM frame. However, this

variant of (S)SLM suffers from error propagation within the receiver-sided descrambler.

In [10], [4], [11], [1] schemes have been proposed, which modulate (either by phase or amplitude) the side information within the information carrying signal and avoid its transmission at all. In [10], [4], [1] the mappings are not performed with discrete phase vectors but continuously distributed ones. The actual applied phase vector is now detected via an maximum-likelihood estimator under the assumption that the information carrying data is taken from a discrete QAM grid. In practical realizations, this scheme is hardly feasible to implement, as continuously distributed phase vectors would disturb the receiver-sided synchronization algorithm. The scheme given in [11] modulates the side information over the whole OFDM frame. Hereby, this scheme suffers from an increase in transmit power. The promising results shown in [11] are only valid for very special choices of the parameters.

B. Insertion of the Side Information

Subsequently, we consider all D subcarriers of one OFDM frame being active. Out of them, D_i (index set \mathcal{I}_i) are allocated for the information carrying signal denoted as the vector \mathbf{A}_i and $D_{\text{si}} = D - D_i$ subcarriers (index set \mathcal{I}_{si} , $\mathcal{I}_i \cap \mathcal{I}_{\text{si}} = \emptyset$) are reserved for the side information denoted as vector $\mathbf{A}_{\text{si}}^{(u^*)}$. For unique representations the number of subcarriers reserved for the side information (index u^*) has to be at least

$$D_{\text{si}} \geq \frac{\lceil \log_2(\hat{U}) \rceil}{\log_2(M)}. \quad (8)$$

The vector representing the side information is generated by a suited mapping \mathcal{S} which maps the indices u of the respective mappings $\mathcal{M}^{(u)}$ to vectors of QAM symbols $\mathbf{A}_{\text{si}}^{(u)}$:

$$\mathcal{S} : u \longrightarrow \mathbf{A}_{\text{si}}^{(u)}, \quad u = 1, \dots, \hat{U}. \quad (9)$$

The insertion of $\mathbf{A}_{\text{si}}^{(u)}$ into the transmit signal (as depicted in the block diagram of Fig. 1) has to ensure that the carriers representing the side information undergo independent fading. Here, we assume that the components of $\mathbf{A}_{\text{si}}^{(u)}$ are distributed uniformly over the whole OFDM frame, i.e., the indices within \mathcal{I}_{si} are chosen that the distance between adjacent subcarriers, reserved for transmission of the side information, is maximized.

All possible signal candidates are now given by

$$\mathbf{A}^{(u)} = [A_d^{(u)}], \quad u = 1, \dots, \hat{U} \quad (10)$$

$$\begin{aligned} \text{with } A_d &= A_{i,d_i}^{(u)} \quad \text{for } d \in \mathcal{I}_i, \quad d_i = 1, \dots, D_i \\ \text{and } A_d &= A_{\text{si},d_{\text{si}}}^{(u)} \quad \text{for } d \in \mathcal{I}_{\text{si}}, \quad d_{\text{si}} = 1, \dots, D_{\text{si}}. \end{aligned}$$

These candidates are transformed into time domain and the “best” one is chosen for transmission as depicted in Fig. 1.

C. Estimation of the Side Information at the Receiver

At the receiver, after application of the discrete Fourier transform and scaling of the carriers (zero-forcing linear equalizer), the received frequency-domain OFDM frame is denoted as the vector \mathbf{Y} which consists of a noisy information carrying part

\mathbf{Y}_i (indices \mathcal{I}_i) and the noisy side information \mathbf{Y}_{si} (indices \mathcal{I}_{si}) according to (10). \mathbf{Y}_{si} is given by the actual transmitted index of side information $\mathbf{A}_{\text{si}}^{(u^*)}$ plus some (complex valued) Gaussian noise with variance $\sigma_n^2/|H_d|^2$.

In order to decode the index u^* a *maximum-a-posteriori* (MAP) estimator [18], [4] is applied. The estimate \hat{u}^* of the index then reads

$$\begin{aligned} \hat{u}^* &= \operatorname{argmax}_{u=1, \dots, \hat{U}} \left\{ \Pr\{\mathbf{A}_{\text{si}}^{(u)} | \mathbf{Y}_{\text{si}}\} \right\} \\ &= \operatorname{argmax}_{u=1, \dots, \hat{U}} \left\{ \prod_{d=0}^{D_{\text{si}}-1} \Pr\{Y_{\text{si},d} | A_{\text{si},d}^{(u)}\} \cdot \Pr\{\mathbf{A}_{\text{si}}^{(u)}\} \right\} \\ &= \operatorname{argmin}_{u=1, \dots, \hat{U}} \left\{ \sum_{d=0}^{D_{\text{si}}-1} \frac{|Y_{\text{si},d} - A_{\text{si},d}^{(u)}|^2}{\sigma_n^2/|H_d|^2} - \log(\Pr\{\mathbf{A}_{\text{si}}^{(u)}\}) \right\}. \end{aligned} \quad (11)$$

To calculate this estimate the a-priori probabilities $\Pr\{\mathbf{A}_{\text{si}}^{(u)}\} = \Pr\{u\}$ have to be known, which can be determined through (5). Moreover, channel state information in terms of the noise variance σ_n^2 and the fading coefficient H_d is required.

In case of SLM where the probabilities $\Pr\{u\}$ are uniformly distributed the term $\log(\Pr\{\mathbf{A}_{\text{si}}^{(u)}\})$ may be dropped, leading to a maximum-likelihood estimator [18].

D. Optimized Mapping of the Side Information

The performance of the estimation of the side information depends strongly on the assignment (mapping \mathcal{S}) of the indices $u = 1, \dots, \hat{U}$ to different signal vectors $\mathbf{A}_{\text{si}}^{(u)}$. Subsequently, we assume that the number of available signals for transmitting the side information is given by $M^{D_{\text{si}}} \geq \hat{U}$.

An optimum mapping \mathcal{S} would be given if the average detection error rate of the index u^* is minimized when applying estimation rule (11). The derivation of such a mapping \mathcal{S} is not trivial. Moreover, the solution depends on the channel state (H_d, σ_n^2) making it infeasible for practical systems. Therefore we optimize the assignment for high signal-to-noise ratios and the assumption that $\sigma_n^2/|H_d|^2$ is (almost) constant over the respective subcarriers. Hence, the sum in the third line of (11) dominates the whole term and, furthermore, $\sigma_n^2/|H_d|^2$ does not influence the minimization criterion. Now, the best strategy is to maximize the pairwise distances of the respective assigned signals $\mathbf{A}_{\text{si}}^{(u)}$, $u = 1, \dots, \hat{U}$.

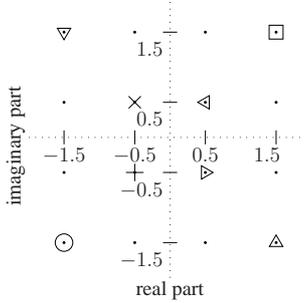
When applying SSLM the distribution of the probabilities $\Pr\{u\}$ is non uniform. In this case indices which occur more often should be better protected against erroneous detections than ones with low probabilities $\Pr\{u\}$. An algorithm which finds a useful mapping \mathcal{S} is given as follows.

First, the set $\mathcal{X} = \{\mathbf{X}^{(l)}\} = \mathcal{A}^{D_{\text{si}}}$, $l = 1, \dots, M^{D_{\text{si}}}$, of all possible signal points from which the symbols $\mathbf{A}_{\text{si}}^{(u)}$ can be drawn is defined. Moreover, it is assumed that all indices u are sorted according to their probabilities in descending order as given in (5) and the assignment is successively performed in this order. The initial signal $\mathbf{A}_{\text{si}}^{(1)}$ is assigned to one outer point of the constellation (e.g., $\mathbf{A}_{\text{si}}^{(1)} = [A_{\text{si},d}^{(1)}]$ with $A_{\text{si},d}^{(1)} = -(\sqrt{M}-1)/2 - j(\sqrt{M}-1)/2$, $d = 1, \dots, D_{\text{si}}$). All remaining signals (indices $u = 2, \dots, \hat{U}$) are assigned

in a greedy manner. In the u^{th} step, the signal $\mathbf{A}_{\text{si}}^{(u)}$ should be allocated to that point within the constellation \mathcal{X} where the minimum distance to all $u - 1$ already assigned signals is maximized. If more points fulfill this condition then all these points are assigned concurrently at this step, namely the points in descending order of their respective probabilities are assigned in descending distance to the initial (most probable) point $\mathbf{A}_{\text{si}}^{(1)}$. The assignment of only one signal, out of the ambiguous ones, would influence the assignment metric for the next steps of the algorithm. Numerical examples have shown that assigning all ambiguous signals from the actual iteration in a single step leads to better results than assigning only a single one and continuing the procedure.

Examples for this assignment with parameters $\hat{U} = 8$ and $M = 16$, $D_{\text{si}} = 1$ or $M = 4$, $D_{\text{si}} = 2$, respectively, are given in Fig. 2. In the top plot of Fig. 2 all $M^{D_{\text{si}}} = 16$ possible signal points are depicted by dots (\cdot). First, the initial signal (\circ) is allocated to the outer point $-1.5 - j1.5$. Next, the second signal (\square) is then assigned to the point with furthest distance. The next two signal points which maximize the minimum distance to the previous allocated ones are the remaining outer points of the constellation. The next points within the signal constellation, whose distance is maximum to the previously assigned ones, are the four inner points ($\pm 0.5 \pm j0.5$) to which the remaining four points are allocated. Noteworthy, the signals are assigned (in order of descending probabilities) to the points with maximum distance to the initial signal (\circ). The example from the bottom plot of Fig. 2 can be developed accordingly.

$M = 16$, $D_{\text{si}} = 1$:



$M = 4$, $D_{\text{si}} = 2$:

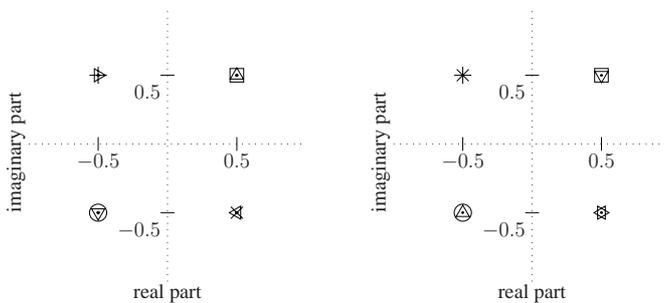


Fig. 2. Example of the mapping \mathcal{S} of the indices u to signal points $\mathbf{A}_{\text{si}}^{(u)}$ for parameters $M = 16$, $D_{\text{si}} = 1$, $\hat{U} = 8$ (top) and $M = 4$, $D_{\text{si}} = 2$, $\hat{U} = 8$ (bottom). The assigned signal points $\mathbf{A}_{\text{si}}^{(u)}$ are depicted as (in descending order of the respective probabilities $\Pr\{U = u\}$): \circ , \square , \triangle , ∇ , \triangleleft , \triangleright , \times , and $+$.

As mentioned above, the assignment rule described in this section is not optimum, as it does not directly minimize the detection error rate of the side information, but an heuristic realization of the mapping \mathcal{S} . However, the good performance of this approach, in terms of error ratios as given in Section. V-B, proves its justification.

V. NUMERICAL RESULTS

Subsequently, we assess the performance of (S)SLM in case of an explicit transmission of the side information. Hereby, an uncoded transmission over an $l_{\text{H}} = 5$ -tap (equal gain) Rayleigh fading channel is considered. The number of subcarriers is chosen to $D = 512$, the cyclic prefix is sufficiently long that no inter-block interferences will occur. As modulation alphabets $M = 4$ and $M = 16$ -QAM are chosen.

A. Complementary Cumulative Distribution of PAR values

Fig. 3 shows the ccdf of PAR of (S)SLM for various values of \hat{U} . It is obvious that both schemes offer significant gains compared to the reference given by the PAR distribution of the original (not PAR reduced) signal. For PAR values less than the threshold PAR_{tol} SLM has better performance than SSLM. However, all values in this region are tolerated whereby deviations in the performance are not critical. For $\text{PAR} > \text{PAR}_{\text{tol}}$ both schemes perform equal. Noteworthy, the huge advantage of SSLM is the significant reduction of computational complexity, as in average only $\mathbb{E}\{U\} \approx 1.5$ signal candidates per OFDM frame have to be assessed.

Moreover, the results are compared with the analytical results (4) and (7) based on the assumption of Gaussian signalling (3). Almost no deviations are recognizable, even though the assumption of statistical independent signal candidates is violated through the insertion of the index representing the side information according to (10).

B. Bit Error Ratio

Fig. 4 shows the bit error ratios of the information carrying part of the signal in case of original SLM and in case of SSLM ($10 \log_{10}(\text{PAR}_{\text{tol}}) = 8.5$ dB). The number D_{si} of reserved subcarriers is set to its minimum possible value according to (8). As reference, we consider the bit error ratio under the assumption of perfect (genie aided) transmission of the index u^* . Significant gains of SSLM compared to original SLM can be observed. Especially for a large number \hat{U} of assessed signal candidates the gap between SSLM and the genie aided curve is hardly visible. Moreover, for larger modulation alphabets (e.g., $M = 16$) the advantages of SSLM are already visible for small numbers \hat{U} . The non-uniform distribution of the index u^* caused by SSLM can hence advantageously be utilized to obtain a very reliable estimate.

In order to quantify the advantages of SSLM also for other values of D_{si} , the differences in required signal-to-noise ratios (SNR) at a maximum tolerable bit error ratio of $\overline{\text{BER}} = 10^{-3}$ between the original SLM or SSLM, respectively and the genie aided case given by

$$\Delta \text{SNR} \stackrel{\text{def}}{=} 10 \log_{10}(\bar{E}_b/N_0) \Big|_{(\text{S})\text{SLM}} - 10 \log_{10}(\bar{E}_b/N_0) \Big|_{\text{genie aided}} \quad (12)$$

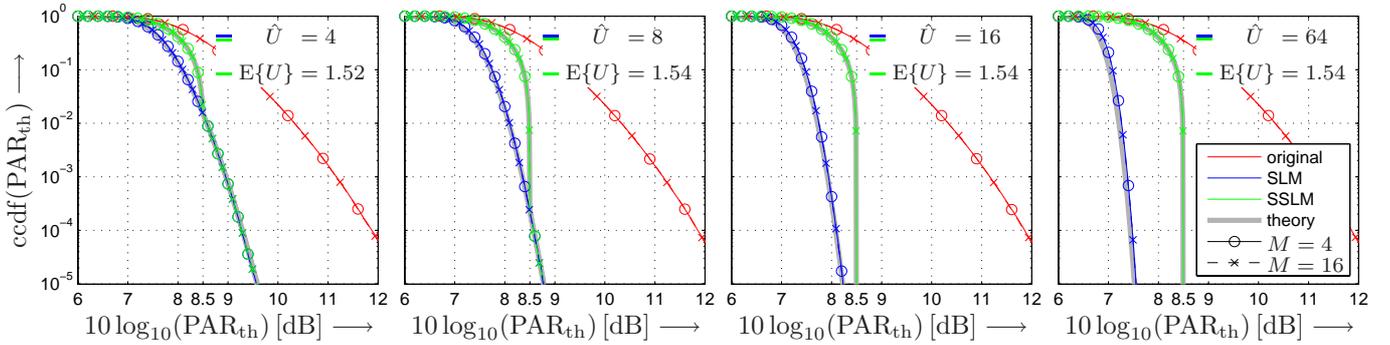


Fig. 3. Ccdf of PAR of the original approach of SLM (blue) and successive SLM with $10 \log_{10}(\text{PAR}_{\text{tol}}) = 8.5$ dB (green); reference: original (not PAR reduced) signal (red); the respective analytical results (4) and (7) based on the assumption of Gaussian signaling (3) are depicted gray; the (maximum) number of assessed signal candidates is chosen to (from left to right) $\hat{U} = 4, 8, 16$, and 64 ; the modulation alphabet is either $M = 4$ -QAM (\circ) or $M = 16$ -QAM (\times); the number D_{si} of subcarriers reserved to transmit the side information is chosen to its minimum value according to (8); $D = 512$.

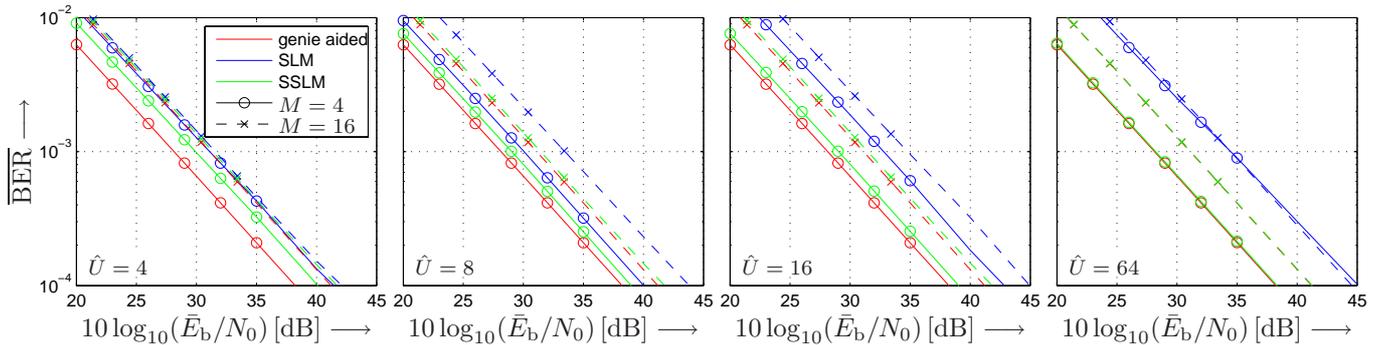


Fig. 4. Average bit error ratio of the information carrying part of the transmitted signal with the original approach of SLM (blue) and successive SLM with $10 \log_{10}(\text{PAR}_{\text{tol}}) = 8.5$ dB (green); reference: bit error ratio for perfect (genie aided) transmission of the side information (red); the (maximum) number of assessed signal candidates is chosen to (from left to right) $\hat{U} = 4, 8, 16$, and 64 ; the modulation alphabet is either $M = 4$ -QAM (\circ) or $M = 16$ -QAM (\times); the number D_{si} of subcarriers reserved to transmit the side information is chosen to its minimum value according to (8), i.e., $M = 4$: $D_{\text{si}} = 1, 2, 2$, and 3 ; $M = 16$: $D_{\text{si}} = 1, 1, 1$, and 2 (from left to right); $l_{\text{H}} = 5$, $D = 512$.

are depicted in Fig. 5. With SSLM the differences are very small and in almost all cases (much) less than 1 dB. For both schemes, SLM and SSLM, increasing the number D_{si} of reserved subcarriers reduces the gap ΔSNR to perfect knowledge of the side information, which converges to 0 dB. This good performance is originated in the fact, that the probability of correct estimation of the side information is near one. Spending more reserved subcarriers than necessary leads to a diversity gain⁴ within the transmission of the side information which is beneficial on the overall error rate. Noteworthy, the convergence of the gap ΔSNR to 0 dB is much faster with SSLM.

Up to now, the threshold of tolerated PAR values for SSLM has always been fixed to $10 \log_{10}(\text{PAR}_{\text{tol}}) = 8.5$ dB. As this choice influences the probabilities of the respective selected signal candidates (5), it has impact on the error performance in case of MAP estimation of the side information. Fig. 6 shows values of ΔSNR for various choices of PAR_{tol} above the critical value $10 \log_{10}(\log(D)) \approx 7.95$ dB. From this plot it can be observed, that allowing larger values of PAR_{tol} will further reduce the loss in error performance of SSLM significantly. However, this step has no impact on the PAR

⁴This diversity gain can be recognized when looking on the ratio of incorrectly estimated indices over the respective SNR. Choosing D_{si} larger than necessary leads to steeper curves.

reduction performance of SLM.

VI. CONCLUSIONS

In this paper, we consider PAR reduction schemes based on multiple signal representation (in particular (S)SLM) and address the issue of transmitting the side information, which is required hereby. We regard the explicit transmission of the side information within the OFDM frame. An appropriate mapping of the side information to reserved carriers is given. The new approach is tailored to the recently proposed extension successive SLM [21], where the probabilities for selecting a certain signal candidate are non-equally distributed. A simple MAP estimation of the side information enables operation near the optimum (genie-aided) error ratios. Moreover, the insertion of the side information into the OFDM frame and an appropriate mapping are discussed. Numerical results show, that with this approach almost no loss in terms of bit error ratios occurs.

However, even if we focus here on the SLM scheme, the simple scheme to transmit and estimate the side information can be used for any related PAR reduction scheme like, e.g., PTS [14].

Recently, extensions of existing PAR reduction techniques to *multiple-input/multiple-output (MIMO)* system have been discussed. In particular extensions of SLM can be found in

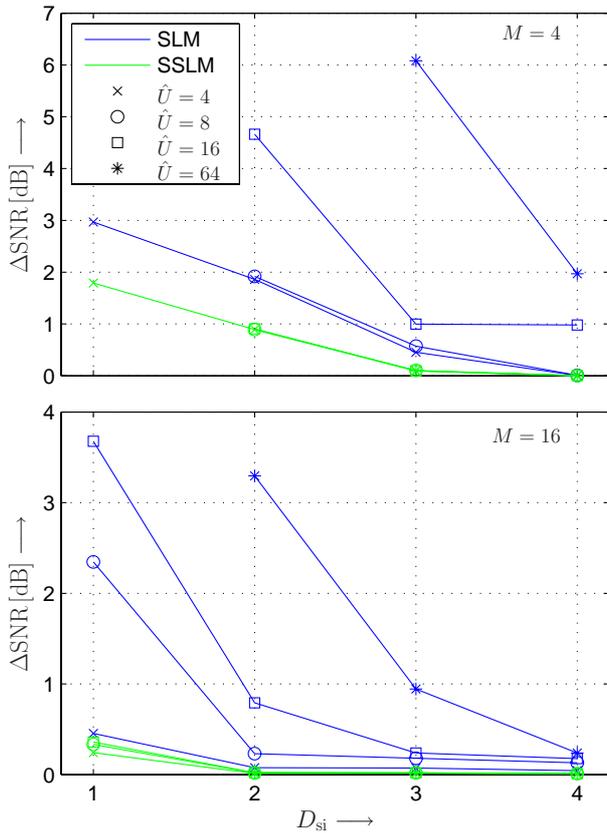


Fig. 5. Difference of required SNR values at a maximum tolerated bit error ratio of $\overline{\text{BER}} = 10^{-3}$ between SLM and perfect knowledge (genie aided) of the side information (blue) and SSLM and perfect knowledge (genie aided) of the side information (green) for various numbers D_{si} of subcarriers reserved to transmit the side information; top: $M = 4$ -QAM; bottom $M = 16$ -QAM; $l_{\text{H}} = 5$, $D = 512$.

[2], [7]. The new approach can be extended to the MIMO case straightforwardly.

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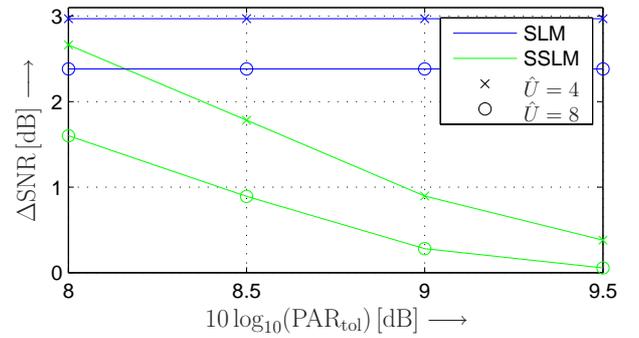


Fig. 6. Difference in required SNR at a maximum tolerated bit error ratio of $\overline{\text{BER}} = 10^{-3}$ values between SLM (with ML estimation) and perfect knowledge (genie aided) of the side information (blue) and SSLM (with MAP estimation) and perfect knowledge (genie aided) of the side information for various tolerated thresholds PAR_{tot} ; the maximum number of assessed signal candidates is $\hat{U} = 4$ with $D_{\text{si}} = 1$ (o) and $\hat{U} = 8$ with $D_{\text{si}} = 2$ (x); $M = 4$ -QAM, $l_{\text{H}} = 5$, $D = 512$.

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