

OUT-OF-BAND POWER REDUCTION IN MIMO OFDM

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ABSTRACT

One of the main disadvantages of orthogonal frequency-division multiplexing (OFDM) is its large peak-to-average power ratio (PAR). In case of nonlinear power amplification this fact causes undesired out-of-band radiation. In this paper PAR reduction schemes for multiple-antenna OFDM, in particular variants of selected mapping (SLM), are considered. Directed SLM (dSLM), where a joint PAR reduction over the antennas is performed, is reviewed and compared to the obvious application of SLM to each single antenna (oSLSM). Due to the avoidance of the signal peaks, less out-of-band radiation is caused. It is shown that both, oSLM and dSLM, provide significant gains in the reduction of the out-of-band radiation compared to conventional OFDM. However, using the same complexity, dSLM provides significant extra gain over oSLM. The gain, which is clearly dependent on the type of nonlinearity and the chosen power backoff, is assessed by means of numerical simulations.

1. INTRODUCTION AND SYSTEM MODEL

Multicarrier modulation, in particular orthogonal frequency-division multiplexing (OFDM), is very popular for transmission over frequency-selective channels [3], and since its invention in the 60th, e.g., [16, 18], it has been used in a number of communication systems and standards. However, for future applications the use of antenna arrays is envisaged leading to parallel OFDM transmission. This approach, often denoted as *MIMO OFDM* (multiple-input/multiple-output), is the most promising candidate for future wireless communication systems.

Throughout this paper, we assume N_T transmit antennas, over which independent data streams are to be communicated. Using an inverse discrete Fourier transform (IDFT) of length D , each of the N_T parallel OFDM transmitters transforms a block of D complex-valued amplitude coefficients (often denoted as “carriers”) $A_{\mu,\nu}$, $\mu = 1, \dots, N_T$, $\nu = 0, \dots, D - 1$, (OFDM frame, vector $\mathbf{A}_\mu = [A_{\mu,0}, \dots, A_{\mu,D-1}]$) into time-domain. As usual in wireless applications, all frequency-domain samples $A_{\mu,\nu}$ are expected to be drawn from the same

constellation with variance σ_a^2 and, for simplicity, all carriers are assumed to be used.

The transmit symbols (equivalent complex-valued baseband signals) are then given as

$$a_{\mu,k} = \frac{1}{\sqrt{D}} \sum_{\nu=0}^{D-1} A_{\mu,\nu} \cdot e^{j2\pi k\nu/D}, \quad \mu = 1, \dots, N_T, \quad k = 0, \dots, D - 1. \quad (1)$$

Defining a time-domain vector $\mathbf{a}_\mu = [a_{\mu,0}, \dots, a_{\mu,D-1}]$, the correspondence is written in short as

$$\mathbf{a}_\mu = \text{IDFT}\{\mathbf{A}_\mu\} = \mathbf{W}_D^H \mathbf{A}_\mu, \quad (2)$$

where \mathbf{W}_D^H is the hermitian (and in the present case inverse) of the Fourier matrix $\mathbf{W}_D \stackrel{\text{def}}{=} \frac{1}{\sqrt{D}} [e^{-j2\pi kl/D}]_{k,l=0:D-1}$ of dimension D .

After introducing a “guard-period” [3], pulse shaping (including digital-to-analog conversion) using a transmit pulse with response $g(t)$ is performed (as it is done in conventional PAM transmission). These complex baseband signals $s_\mu(t)$ are finally modulated to radio frequency and radiated from the antennas.

Due to the superposition of the individual signal components (the carriers), the OFDM time-domain samples $a_{\mu,k}$ are almost Gaussian distributed and hence exhibit a large *peak-to-average power ratio* (PAR)¹

$$\text{PAR}_\mu \stackrel{\text{def}}{=} \frac{\max_k |a_{\mu,k}|^2}{\text{E}\{|a_{\mu,k}|^2\}} = \frac{\max_k |a_{\mu,k}|^2}{\sigma_a^2}. \quad (3)$$

This fact significantly complicates implementation of the radio frequency frontend since amplifiers operating linearly over a wide amplitude range have to be used. Nonlinear distortion and clipping of the transmit signals $s_\mu(t)$ lead to a loss in error performance and—even worse—undesired out-of-band radiation. Hence, in order to avoid out-of-band radiation, the PAR of all N_T transmit signals should be simultaneously as small as possible. Performance is governed by the worst-case PAR, and we consider

$$\text{PAR} \stackrel{\text{def}}{=} \max_{\mu=1, \dots, N_T} \text{PAR}_\mu = \frac{\max_{\mu,k} |a_{\mu,k}|^2}{\sigma_a^2}. \quad (4)$$

¹We first restrict the discussion to the PAR of the discrete-time samples $a_{\mu,k}$.

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In literature, a variety of PAR reduction techniques for (single-antenna) OFDM are known—for a brief overview see, e.g., [7]. One of the most prominent approaches is to use *redundant signal representations*. Information to be transmitted is encoded/mapped in different versions; the representative with the lowest PAR (or any other criterion) is selected and actually transmitted. This PAR reduction scheme is called *selected mapping (SLM)* [2, 12, 4].

Almost all PAR reduction techniques were designed for single-antenna transmission. Meanwhile a few extensions to MIMO OFDM are discussed in literature, e.g., [10, 1, 8, 19, 9, 5, 6].

In this paper, we study PAR reduction in MIMO OFDM and its effect on the out-of-band radiation. In particular, a recently introduced generalization of selected mapping, called *directed SLM (dSLM)* [5], is considered. Section 2 reviews PAR reduction for MIMO OFDM and discusses on which signals (discrete-time/continuous-time) the selection process should be based for best performance at lowest complexity. In Section 3, the reduction of the signal peaks and the out-of-band radiation when using SLM is assessed by mean of numerical simulations. Some conclusions are drawn in Section 4.

2. PAR REDUCTION IN MIMO OFDM

In this section we review SLM for single-antenna OFDM and an extension to multiple-antenna transmission. The criterion for selecting the best representative is discussed.

2.1. Single-Antenna SLM

In selected mapping each OFDM frame is *mapped* to a number of U (independent) candidates representing the same information. From these that one with the lowest PAR (or any other criteria) is *selected* [2, 12, 4].

A first approach to generate these candidates is to multiply carrier-wise the original OFDM frame \mathbf{A} by U phase vectors $\mathbf{P}^{(u)} \stackrel{\text{def}}{=} [P_0^{(u)}, \dots, P_{D-1}^{(u)}]$, $u = 1, \dots, U$, $P_\nu^{(u)} = e^{j\varphi_\nu^{(u)}}$. These phase vectors are randomly selected when designing the system and known to both transmitter and receiver. Favorably, $\varphi_\nu^{(u)}$ is chosen from $\{0, \pi/2, \pi, 3\pi/2\}$; in this case only pure inversion and/or interchange of the quadrature components have to be performed [2]. Moreover, the QAM constellations in each carrier are invariant to rotations by multiples of $\pi/2$ and hence all other components of the OFDM system, in particular synchronization, are not affected.

The candidates are transformed into time-domain, $\mathbf{a}^{(u)} = \text{IDFT}\{\mathbf{A} \odot \mathbf{P}^{(u)}\}$ (\odot denotes element-wise multiplication), their PARs are calculated, and the “best” OFDM frame $\mathbf{a}^{(u^*)}$ is actually transmitted. Fig. 1 sketches SLM for single-antenna transmission.

In order to recover data, for this variant of SLM side information (the index u^*) has to be communicated to the re-

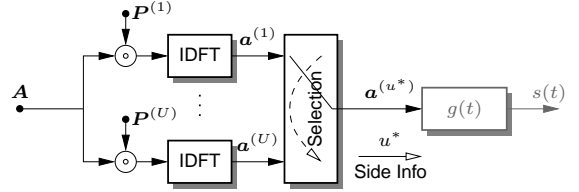


Fig. 1. SLM for single-antenna transmission (including pulse shaping for generation of the continuous-time transmit signal). Variant using phase vectors.

ceiver. This side information has to be incorporated into the OFDM frame (which may affect PAR) and particularly protected against error. However, using the variant of SLM presented in [4] no explicit side information at all has to be communicated. Here, the candidates are generated by prefixing the binary data with a label and scrambling this sequence (pure recursive filtering over the binary field). Then the candidates using different labels are mapped onto the frequency-domain symbols A_ν . Transformation to time-domain and selection is done as for the above version of SLM. At the receiver, an inverse scrambler suffices to separate label (this prefix is simply ignored) and data.

Since both variants of SLM perform the same and all subsequent discussions are equally valid, we restrict to the somewhat conceptually simpler variant with phase modification. However, in practice the scrambler variant is preferable since no side information has to be recovered at the receiver.

2.2. MIMO Extensions of SLM

In [1], SLM is individually applied to each of the N_T parallel schemes in MIMO OFDM, and called *ordinary SLM (oSLM)*. This approach is depicted in Fig 2. It can be shown that the

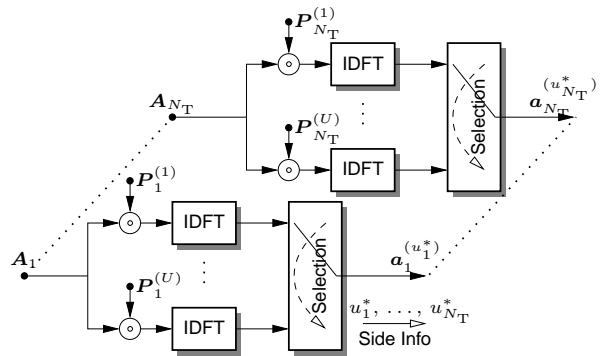


Fig. 2. Ordinary SLM for multi-antenna transmission.

statistics of PAR of conventional MIMO OFDM and oSLM are both worse by the factor N_T compared to single-antenna SLM [1, 6]

Besides oSLM, in [1] a MIMO version of SLM called *simplified SLM (sSLM)* is introduced. Here, the phase vec-

tors are not individually chosen per antenna, but the same for all antennas. This approach requires less side information as only a single index has to be communicated. However, no gain in complexity is achieved and performance of this PAR reduction schemes is significantly worse. Hence, we do not consider sSLM in the sequel.

It is well-known that MIMO transmission offers advantages compared to single-antenna schemes. In particular, the diversity order (slope of the error rate curve in double-logarithmic scale) may be as large as $\min\{N_T, N_R\}$, where N_R is the number of receive antennas and usually $N_R \geq N_T$. These advantages of MIMO transmission are similarly utilizable for PAR reduction. In [5, 6] a scheme called *directed SLM* (*dSLM*) was presented which indeed exploits the potential of MIMO transmission. As a result, the statistics of PAR shows a larger slope, comparable to the diversity gain with respect to error rate.

Main idea of dSLM is to invest complexity only where PAR reduction is really needed—instead of performing U trials individually for each of the N_T transmitters, the budget of $N_T U$ IDFTs is used to successively improve the currently highest PAR over the antennas. In the first step, the PAR of the N_T initial OFDM frames is calculated ($\mathbf{P} = [1, \dots, 1]$). Then, in each of the $N_T(U - 1)$ successive steps, the OFDM frame with instantaneously highest PAR is considered and using a new phase vector $\mathbf{P}^{(u)}$, a reduction of PAR is tried. The pseudo code of the algorithm is depicted in Fig. 3. Of course, instead of using phase vectors for generating the candidates, the scrambler variant of SLM can be used as well.

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given:  $U, [\mathbf{P}^{(1)}, \dots, \mathbf{P}^{(N_T(U-1))}]$ 
function  $[\mathbf{a}_1, \dots, \mathbf{a}_{N_T}] = \text{dSLM}([\mathbf{A}_1, \dots, \mathbf{A}_{N_T}])$ 
1   $\mathbf{a}_\mu = \text{IDFT}\{\mathbf{A}_\mu\}$ , calc.  $\text{PAR}_\mu$ ,  $\mu = 1, \dots, N_T$ 
2  for  $u = 1, \dots, N_T(U - 1)$ 
3     $[\text{PAR}_{\max}, \mu_{\max}] = \max\{\text{PAR}_1, \dots, \text{PAR}_{N_T}\}$ 
4     $\mathbf{a}_{\text{new}} = \text{IDFT}\{\mathbf{A}_{\mu_{\max}} \odot \mathbf{P}^{(u)}\}$ , calc.  $\text{PAR}_{\text{new}}$ 
5    if ( $\text{PAR}_{\text{new}} < \text{PAR}_{\mu_{\max}}$ )
6       $\mathbf{a}_{\mu_{\max}} = \mathbf{a}_{\text{new}}$ ,  $\text{PAR}_{\mu_{\max}} = \text{PAR}_{\text{new}}$ 
7    endif
8  endfor

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Fig. 3. Pseudocode of directed selected mapping for MIMO OFDM (variant using phase vectors). The function \max returns the maximum and the corresponding index, and \odot denotes element-wise multiplication.

2.3. Selection Criteria

Up to now, we have restricted the discussion to the PAR of the discrete-time symbols at the output of an IDFT of length D , equal to the number of carriers. However, the continuous-time transmit signal, which has to be power amplified, is obtained after performing pulse shaping, i.e., filtering the sequence of time-domain samples $a_{\mu,k}$ (with chip duration $T_c = T/D$, if T denotes the duration of an OFDM frame) with the impulse

response $g(t)$, cf. Fig. 1. Due to pulse shaping, and dependent of $g(t)$, the PAR of the continuous-time signal can be significantly larger than that of the discrete-time samples. An exact analytic correspondence between both PARs is not known but some bounds are available in literature, e.g., [20, 17, 11].

If the power amplifier is overdriven, i.e., input signals outside the range of (almost) linear operation are present, out-of-band radiation is generated. Hence, in order to avoid out-of-band radiation, peak power of $s(t)$ has to be limited to the linear range of the amplifier. Using PAR reduction schemes, the probability of exceeding a given threshold can be (significantly) reduced. In turn, lower out-of-band radiation is generated or it is even avoided.

The above PAR reduction schemes can immediately be modified in order to base selection on the continuous-time transmit signals $s_\mu(t)$ rather than the samples $a_{\mu,k}$. The obvious modification is shown in Fig. 4 for one antenna out of the N_T .

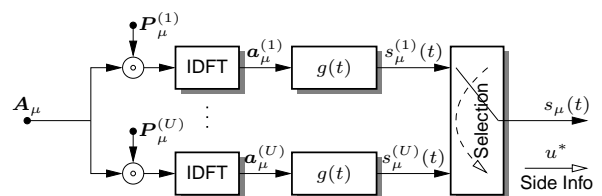


Fig. 4. Modification of SLM for controlling the continuous-time transmit signal.

Unfortunately, the above modification increases complexity significantly, as per antenna U times pulse shaping has to be performed. However, thereby not the entire continuous-time signal has to be generated, but a sufficiently large number of samples. Using some oversampling factor O , instead of considering D (discrete-time) samples $a_{\mu,k}$, a number of $O \cdot D$ samples of $s_\mu(t)$ are assessed in the selection process. Usually, $O = 4, \dots, 8$ suffices.

IDFT and filtering may be interchanged, leading to savings in complexity. Instead of filtering (convolution) in time-domain, multiplication in frequency-domain is performed, as it is done in fast convolution [13]. For that, $\mathbf{A}_\mu^{(u)}$ is repeated O times, multiplied with samples of the transfer function corresponding to $g(t)$ and then, using a DFT of length $O \cdot D$, transformed into time-domain. However, following this approach, not the actual continuous-time signal—which is given by the *linear* convolution—is obtained, but the *cyclic* convolution is carried out. Since in OFDM a guard interval is introduced at the transmitter and hence at least some partial cyclic convolution is performed anyway, the selection is not based on the actual transmit signals but good approximations of them.

Following the above discussion, a substitute for the transmit signal can be used in the selection process, which is even simpler to calculate. Performing an IDFT of length $O \cdot D$ on \mathbf{A}_μ , an oversampled version of \mathbf{a}_μ is obtained [13]. Now,

not the actual shape of $g(t)$ is taken into account, but an ideal low-pass filter is implicitly assumed. If the roll-off of $g(t)$ is not too large, i.e., steep band edges are present, this approximation may be sufficient.

3. NUMERICAL RESULTS AND DISCUSSION

The performance of the MIMO versions of SLM are now assessed by means of numerical simulations. Thereby we are particularly interested in the reduction of the out-of-band radiation but also have to consider the distribution of the PAR.

Unless otherwise stated, the number of carriers (all used) is $D = 512$, the modulation in each carrier is 4PSK, and transmission of $N_T = 4$ parallel data streams is assumed. The phase vectors— $U = 4$ candidates (per antenna) are used—are chosen randomly and the phases are restricted to the above given set of rotations by multiples of $\pi/2$. For pulse shaping a raised cosine pulse [14] with roll-off factor 0.3 is assumed.

3.1. Distribution of PAR

First, we study the probability that the PAR of an OFDM frame exceeds a given threshold PAR_0 , i.e., we consider the *complementary cumulative distribution function (ccdf)*

$$\text{ccdf}_{\text{PAR}}(\text{PAR}_0) = \Pr\{\text{PAR} > \text{PAR}_0\} \quad (5)$$

of the PAR ($\Pr\{\cdot\}$: probability). The ccdfs of the worst-case PAR for oSLM and dSLM with $U = 4$ and, additionally, $U = 16$ candidates are compiled in Fig. 5. It is worth noting that the PAR curves for dSLM exhibit a larger slope than for oSLM. Their slope is the same as (single-antenna) SLM with U candidates. In dSLM, a slope (almost) corresponding to $N_T U$ candidates is achieved.

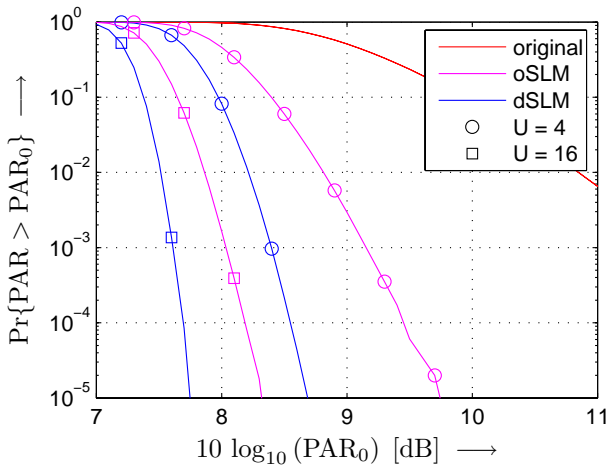


Fig. 5. Ccdf of PAR of the discrete-time symbols $a_{\mu,k}$ for MIMO OFDM without PAR reduction and with oSLM and dSLM. $D = 512$, $N_T = 4$, 4PSK, $U = 4, 16$ candidates.

In Fig. 6 the ccdf of the (worst-case) PAR of the continuous-time transmit signals is plotted. Thereby, for calculation the actual PAR of the OFDM frame an oversampling factor $O = 8$ is used, i.e., per OFDM frame $O \cdot D$ time-domain samples are considered. Noteworthy, the interference due to pulse shaping between subsequently transmitted OFDM frames and the influence of the guard interval are not taken into account in the PAR calculation.

In the top left part of Fig. 6 the situation is plotted when PAR reduction is based on the discrete-time symbols. Compared to Fig. 5 a somewhat higher PAR is present (notice the different range of the x -axis) since the crest factor of the pulse shape $g(t)$ is now included. Compared to original OFDM, only a small gain is achieved by oSLM and dSLM; here both approaches perform almost the same.

The situation changes when selection in SLM is based on an oversampled version (IDFT of length $O \cdot D$), middle left ($O = 2$) and right ($O = 8$) of Fig. 6. Here, oSLM and dSLM provide a larger gain at the cost of increased complexity. dSLM clearly shows an advantage over oSLM. The best results are achieved when the actual continuous-time transmit signal is considered in the selection process, top right of Fig. 6. Almost the same performance is achieved when weighting with the transfer function corresponding to $g(t)$ is done (cyclic convolution), bottom left ($O = 2$) and right ($O = 8$). Using these approaches, dSLM shows the largest gain over original OFDM and oSLM. Since multiplication has only to be done within the roll-off regions of the filter, complexity in the PAR reduction algorithm is mainly governed by the repeated calculation of IDFTs of length $O \cdot D$.

In summary, in order to fully obtain the gains of dSLM over conventional OFDM and oSLM, PAR should be calculated by appropriate weighting and oversampling, where $O = 2$ already offers very good performance.

3.2. Power Spectral Densities and Out-of-Band Radiation

The effect of nonlinear power amplification of the MIMO OFDM transmit signals on the out-of-band radiation is now assessed. For that, a simple soft-limiting nonlinearity according to

$$s_L(t) = \begin{cases} s(t), & |s(t)| < s_{\max} \\ s_{\max} \cdot \frac{s(t)}{|s(t)|}, & |s(t)| \geq s_{\max} \end{cases} \quad (6)$$

is assumed, i.e., the amplitude characteristic is ideally linear up to the perfectly horizontal saturation at s_{\max} ; the signal phase is not affected (AM/AM model). The *power backoff* (PBO, clipping level compared to average power) is defined as

$$\text{PBO} = \frac{s_{\max}^2}{\mathbb{E}\{|s(t)|^2\}}. \quad (7)$$

and again the raised cosine pulse shape with roll-off factor 0.3 is assumed.

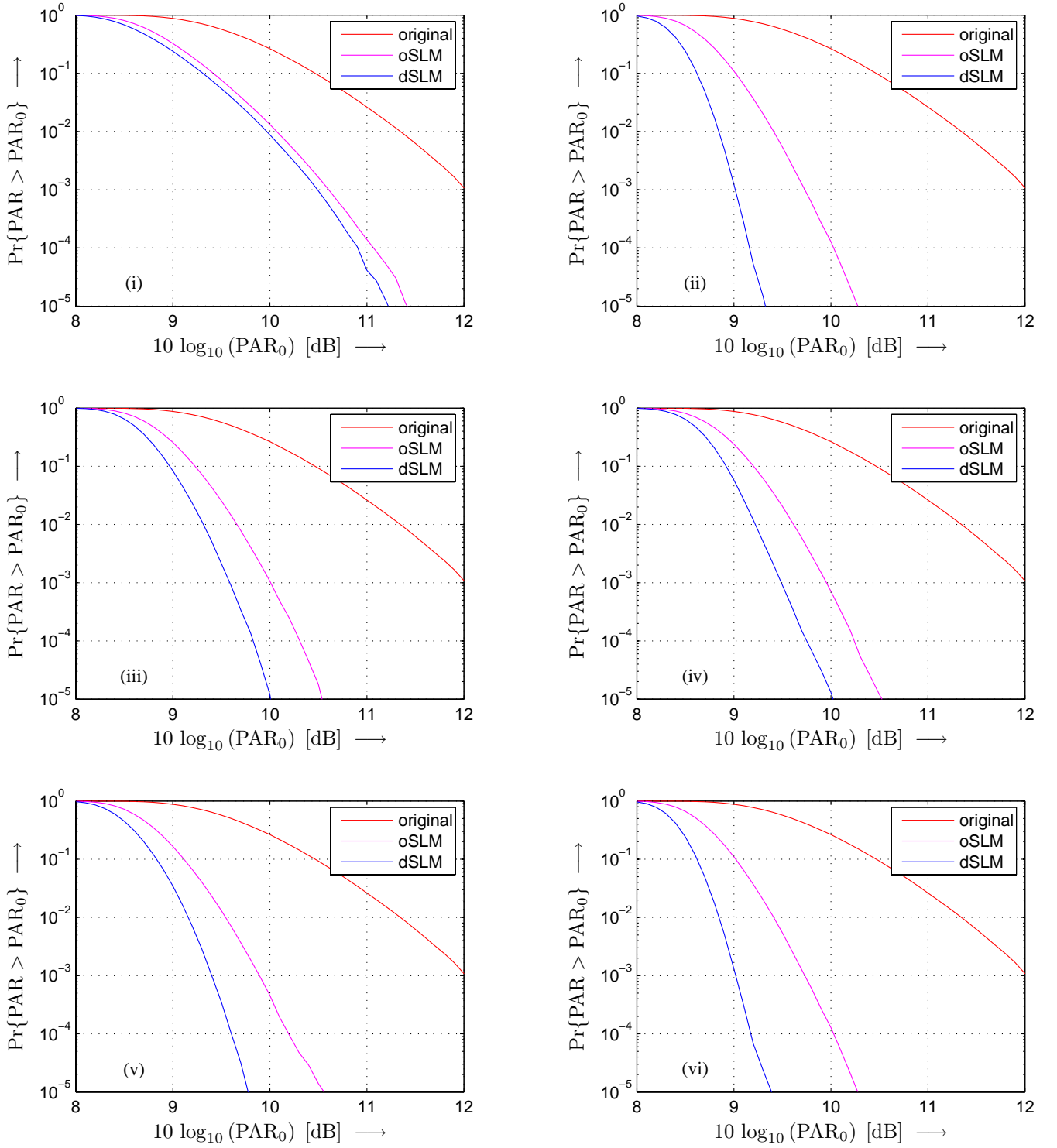


Fig. 6. Ccdf of PAR of the continuous-time transmit signals $s_\mu(t)$ for MIMO OFDM without PAR reduction and with oSLM and dSLM. $D = 512$, $N_T = 4$, 4PSK, $U = 4$ candidates. Selection in PAR reduction based on (i) discrete-time symbols, (ii) continuous-time (oversampling factor $O = 8$) transmit signal, (iii, iv) oversampled ($O = 2, 8$) discrete-time symbols, (v, vi) oversampling ($O = 2, 8$) and weighting in frequency-domain (cyclic convolution).

The impact of PAR reduction on the out-of-band radiation is illustrated in Fig. 7, where the average (averaged over all antennas and realizations of OFDM frames) power spectral density is plotted over the normalized (chip duration T_c) frequency. Here, selection in PAR reduction is done based on the continuous-time transmit signal after pulse shaping ($O = 8$ is again used) but before the nonlinearity.

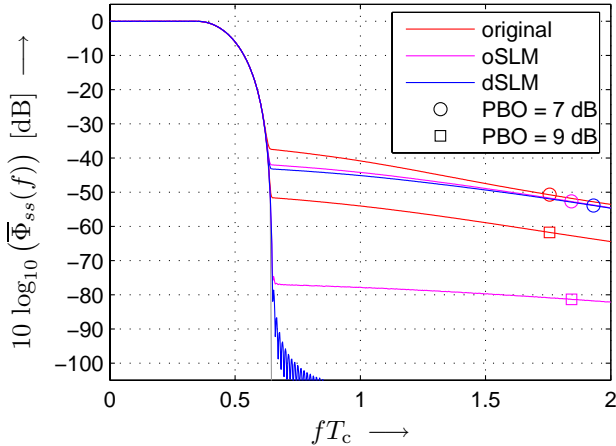


Fig. 7. Average power spectral density for MIMO OFDM without PAR reduction and with oSLM and dSLM. $D = 512$, $N_T = 4$, 4PSK, $U = 4$ candidates. PBO = 7 and 9 dB. Gray: ideal power spectral density. Selection in PAR reduction based on the continuous-time ($O = 8$) transmit signal.

For a PBO of 7 dB, significant out-of-band radiation occurs and no improvement by PAR reduction is possible. This is consistent with the ccdf curve in Fig. 6—for a PAR of 7 dB, all ccdf curves have merged. Using a PBO of 9 dB, original MIMO OFDM still produces significant out-of-band radiation, whereas using oSLM a reduction by approx. 25 dB is possible. When using dSLM, a further huge gain by almost 30 dB over oSLM (more than 50 dB over original MIMO OFDM) is possible. Noteworthy, this gain is achieved without additional complexity compared to oSLM, as both versions of SLM use the same number of candidates and hence PAR calculations.

For comparison, the PSDs after soft limitation are shown in Fig. 8 when selection in the PAR reduction scheme is based on the oversampled discrete-time symbols without (top row) and with (bottom row) weighting in frequency-domain. The out-of-band radiation is not reduced that much as above but still significant gains (up to 40 dB at PBO = 9 dB and $O = 2$) compared to conventional OFDM are achievable. Especially when using weighting in frequency domain, very good performance is observable. For fully exploiting the potential of dSLM, an oversampling factor of $O = 2$ seems not to suffice; however an improvement of more than 10 dB compared to oSLM (same complexity) is possible. With $O = 8$ an even better reduction of the out-of-band radiation is possible which

is very close to that when considering the continuous-time transmit signal in the selection process.

Finally, other types of nonlinearities are considered. In particular the “Rapp model” [15], which is defined according to

$$s_L(t) = \frac{s(t)}{\left(1 + \left(\frac{|s(t)|}{s_{\max}}\right)^{2p}\right)^{\frac{1}{2p}}} \quad (8)$$

is employed. The parameter p is chosen to $p = 3$, which reflects a heavily nonlinear model, and $p = 10$, which is moderately distorting. For $p \rightarrow \infty$, the Rapp model tends to the above used soft limiter.

The PSDs after the nonlinearity (8) are plotted in Fig. 9. The selection in the PAR reduction algorithm is based on oversampling ($O = 8$) and weighting in frequency-domain. Compared to Fig. 8 it is visible that these two types of nonlinearities cause more distortion than the above soft limiter. In particular, for $p = 3$, where even signals with small amplitudes are already distorted, almost no improvement of the out-of-band radiation can be achieved for the shown power backoffs of 9 and 10 dB. For $p = 10$ oSLM and dSLM provide significant gains (15 to 20 dB) over conventional OFDM. Spending the same complexity, dSLM again offers an extra gain of approximately 5 dB.

4. CONCLUSIONS

In this paper PAR reduction schemes for MIMO OFDM, in particular variants of selected mapping, have been considered. It has been shown that dSLM, recently introduced in literature, provides a better reduction of the signal peaks than the obvious application of single-antenna SLM to each of the N_T antennas (oSLM). In turn, due to the avoidance of the signal peaks, less out-of-band radiation is caused in nonlinear power amplifiers.

In summary, it can be stated that PAR reduction based on the SLM principle can reduce out-of-band radiation significantly in OFDM transmission with nonlinear power amplification. The gain is clearly dependent on the type of nonlinearity (higher gain if the nonlinearity is sharply limiting peaks, lower gains if already small amplitudes are distorted) and the power backoff (higher gains for higher PBO). In all cases, dSLM—a joint PAR reduction over the antennas—offers an extra gain over oSLM, where each antenna is treated separately.

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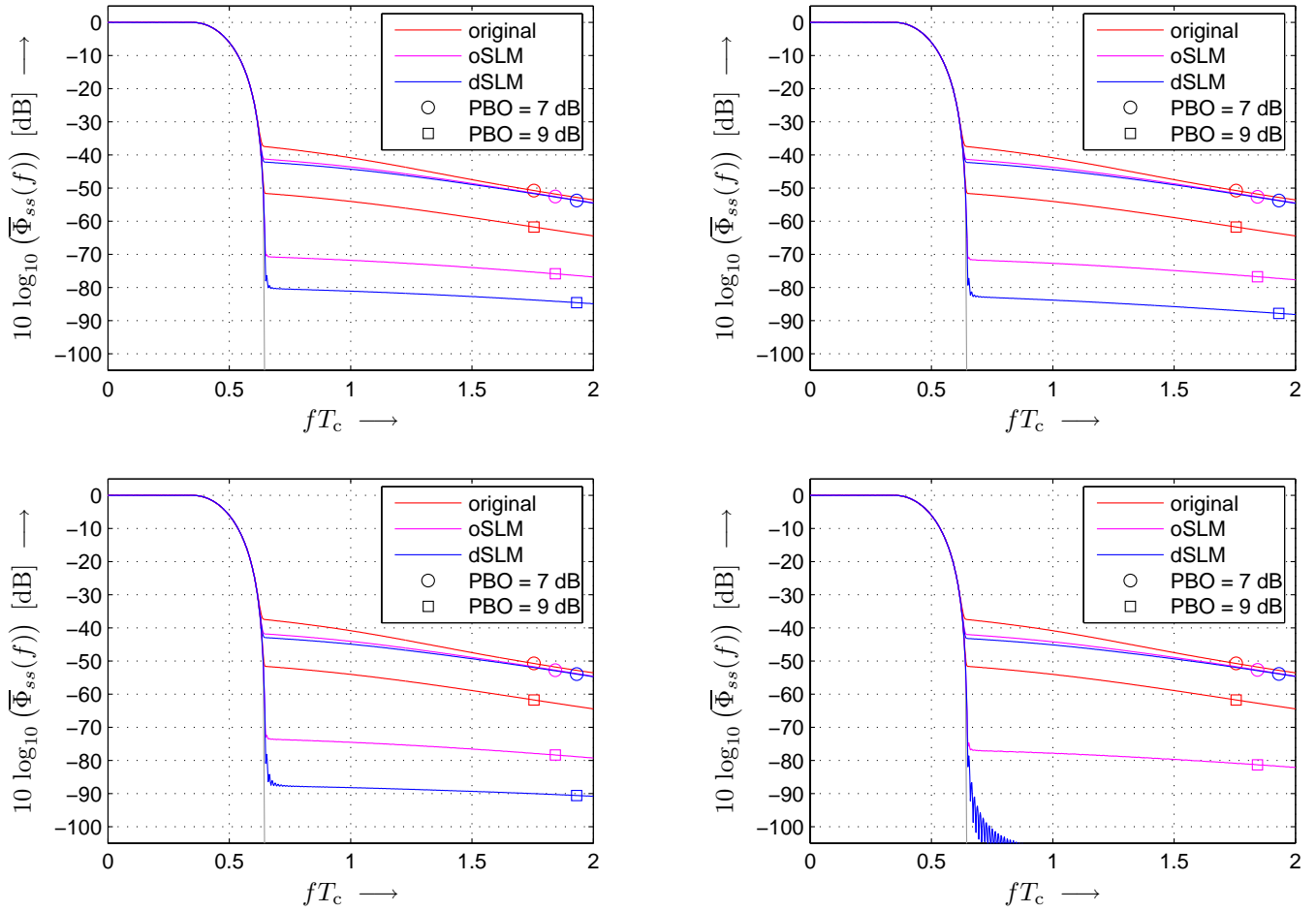


Fig. 8. Average power spectral density for MIMO OFDM without PAR reduction and with oSLM and dSLM. $D = 512$, $N_T = 4$, 4PSK, $U = 4$ candidates. PBO = 7 and 9 dB. Gray: ideal power spectral density. Selection in PAR reduction based on Top: oversampled discrete-time symbols; Bottom: oversampling and weighting in frequency-domain (cyclic convolution). Left: $O = 2$, Right: $O = 8$.

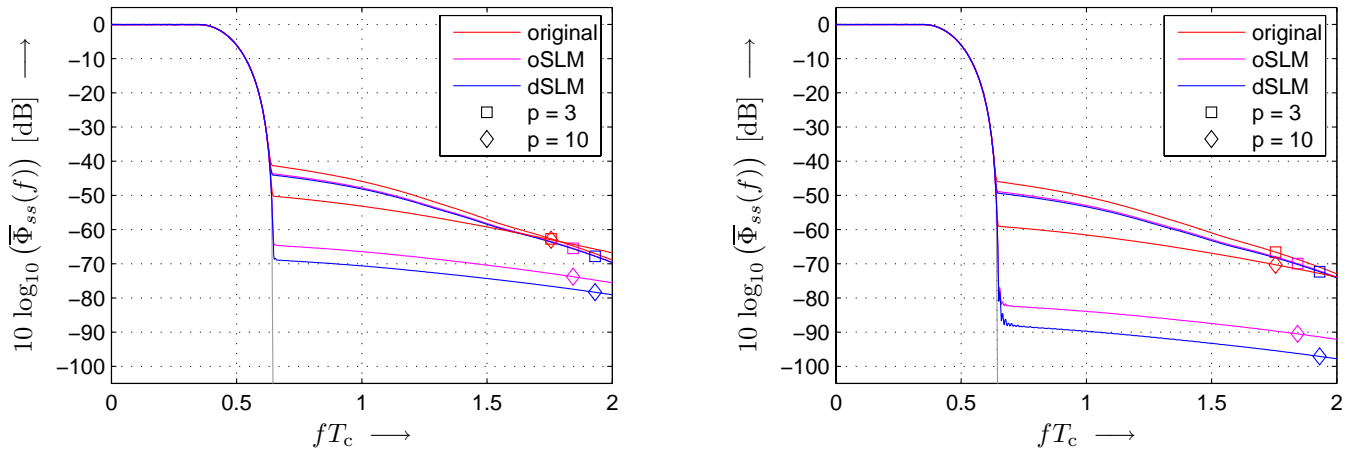


Fig. 9. Average power spectral density for MIMO OFDM without PAR reduction and with oSLM and dSLM. $D = 512$, $N_T = 4$, 4PSK, $U = 4$ candidates. Gray: ideal power spectral density. Selection in PAR reduction based on oversampling ($O = 8$) and weighting in frequency-domain (cyclic convolution). Left: PBO = 9, Right: 10 dB.

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